Mathematics Across The Curriculum
at Dartmouth College

Geometry in Art & Architecture

by Paul Calter

A dynamic branch of mathematics, geometry also serves as a creative tool for engineers, artists, and architects. Squaring the Circle: Geometry in Art and Architecture is the first textbook to cover both art and geometry extensively. The text's wide-ranging exercise sets and related projects allow students to practice and master the mathematics presented. Each chapter introduces mathematical concepts geometrically and illustrates their nontraditional applications in art and architecture throughout the centuries.

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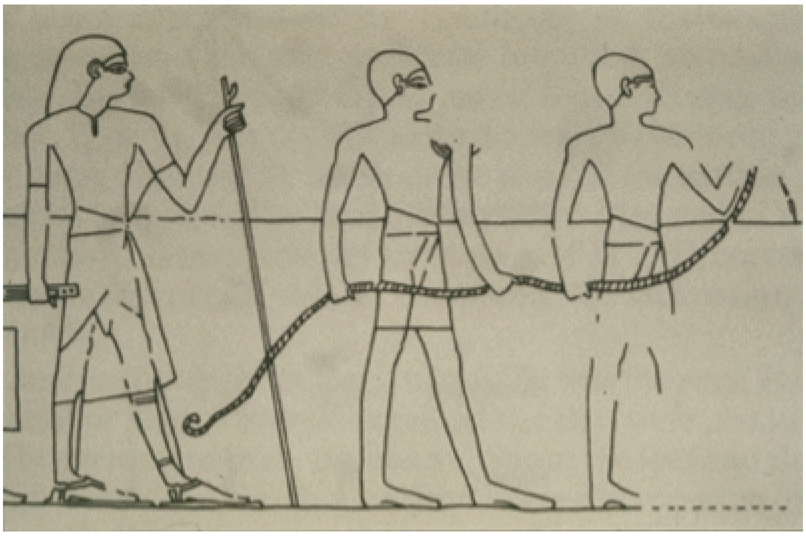
INTRODUCTION

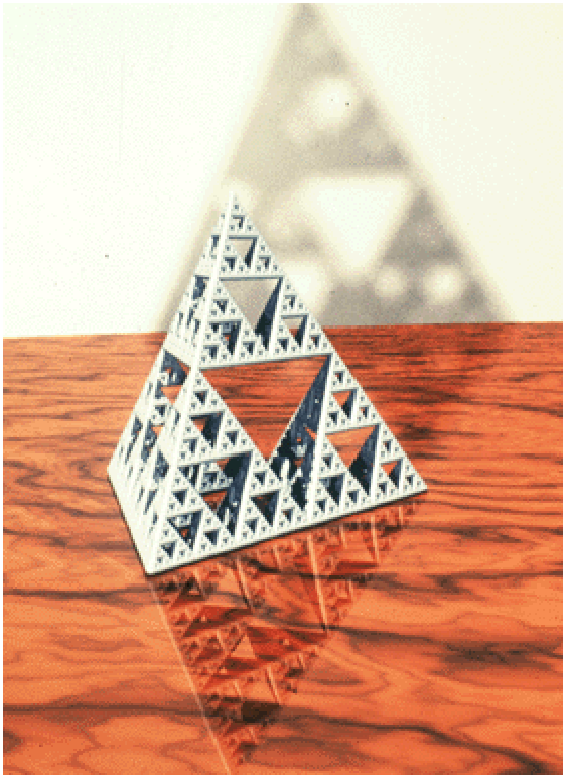


Egyptian Rope Stretchers
Tompkins, Peter. *Secrets of the Great Pyramid.* NY: Harper, 1971. p. 22.

Welcome to **Geometry in Art & Architecture.** We'll be taking a long journey together, starting in Egypt, like the travellers in this picture. There, the story goes, geometry got its start when rope stretchers were sent out to put back the boundary markers washed away by the Nile.

In addition to looking at art and architecture, we'll cover any mathematics-related topics as we go along. The Math Topics for the first unit will be an introduction to the triangle in general, and the so-called Egyptian triangle, contained in the great pyramid. Since the Egyptian triangle contains the golden ratio, we'll introduce the ideas of ratio and proportion here, and for squaring of the circle, we must be able to find perimeters and areas of the square and the circle.

The plan is to go more or less chronologically, following threads of Art, Mathematics, and Architecture, from Egypt to the present.

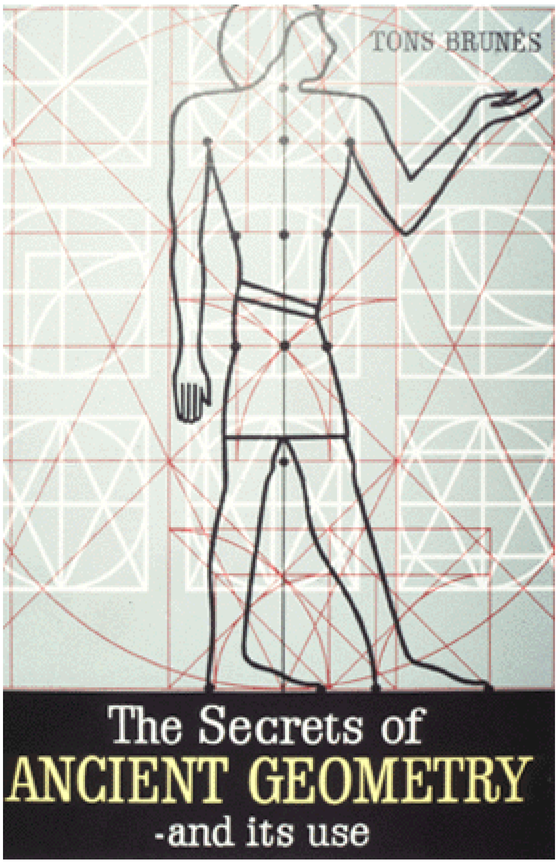


Fractal Tetrahedron

We'll start our journey with a pyramid, and we'll also end with a very different pyramid, a Sierpinski tetrahedron, in our final unit on Chaos and Fractals.

We'll limit ourselves to Western art only, but even with that restriction the coverage is very wide. That means, of course, that we can't go too deeply into any one topic.

Things We'll Look For



Brunes, Tons, *The Secrets of Ancient Geometry - and its use.* Copenhagen. Rhodos, 1967.

Brune's Cover

As we go through the material we'll be looking for:

1. **Proportions between the parts** of a building, a painting, or a sculpture. In particular, we'll look for the golden ratio and the musical ratios.
2. **Use of geometric symbols**, such as the circle, mandala, triangle, square, pentagram, hexagon, or octagon, and their use in so-called Sacred Geometry.
3. **Geometric Constructions**, like squaring the circle, the Gothic Master Diagram, the sacred cut, and constructions of the pentagon.
4. **Shapes of Frames**; how they are chosen and how they affect the contents of a painting.
5. **Art Motifs**, especially recurring themes that we see over and over in art.
6. **Math content**; any geometry or other math that is closely related to the art or architecture we're studying.
7. **People**, ones that played a key role in developing the ideas related to this course, and especially those that were both mathematicians and artists or architects.

Skeptical Attitude

We'll see that writers in this field sometimes make unsupported claims. Rudolf Wittkower, in his *Architecture in the Age of Humanism* says

". . . in trying to prove that a system of proportions has been deliberately applied . . . one is easily misled into finding . . . those ratios which one sets out to find. Compasses in the scholar's hand do not revolt."

In other words, we tend to find what we're looking for, whether its there or not. We will hope to avoid that pitfall by questioning everything.

Mathematics Across the Curriculum

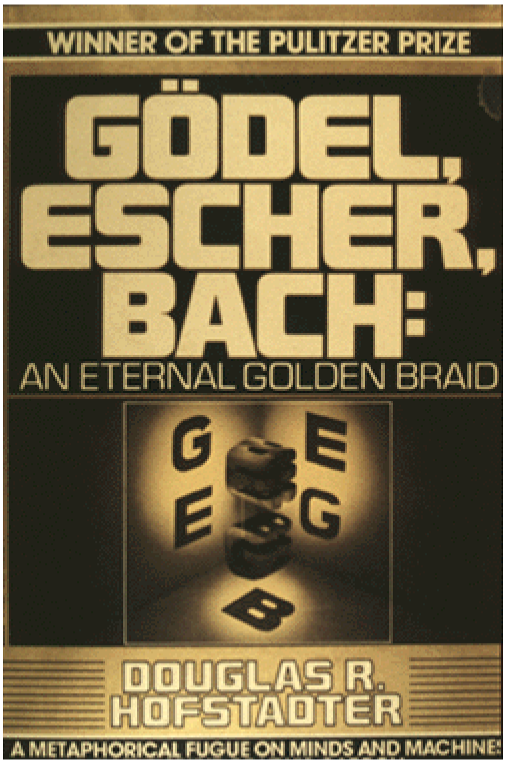
MATC Logo

This course is one of several developed under a grant from the National Science Foundation to Dartmouth, called *Mathematics Across the Curriculum.* Some courses being developed at Dartmouth are ones that try to integrate math with:

physics and chemistry
textile design
psychology and medicine
music
earth sciences
Renaissance thought

and this one, combining math with art and architecture.

The Eternal Golden Braid



Hofstadter, Douglas. Gödel, Escher, Bach: an Eternal Golden Braid. NY: Vintage, 1979. Hofstadter Cover

In *"Gödel, Escher, Bach:",* Douglas Hostadter says

"I have sought to weave an eternal golden braid out of these three strands, Gödel, Escher, Bach, a mathematician, an artist, and a composer."

In other words, math, art, and music. In this course we hope to trace just two strands of his eternal golden braid, art (and architecture) and math, and sometimes connect them with strands from literature, mythology, and religion .

We've planned an exciting journey, to follow these strands over 5000 years and several continents, and we really hope that you'll join us for the trip!

The Golden Ratio & Squaring the Circle in the Great Pyramid

"Twenty years were spent in erecting the pyramid itself: of this, which is square, each face is eight plethra, and the height is the same; it is composed of polished stones, and jointed with the greatest exactness; none of the stones are less than thirty feet." -Heroditus, Chap. II, para. 124.

Slide 2-1: The Giza Pyramids and Sphinx as depicted in 1610, showing European travelers Tompkins, Peter. *Secrets of the Great Pyramid.* NY: Harper, 1971. p. 22

The Great Pyramid

Slide 2-2: The Great Pyramid of Cheops

Tompkins, Peter. Secrets of the Great Pyramid. NY: Harper, 1971. p. 205

We start our task of showing the connections between geometry, art, and architecture with what appears to be an obvious example; the pyramids, works of architecture that are also basic geometric figures.

The pyramids were built in the lifetime of a single king, and were to help him in become immortal. They were made mostly in 4th dynasty of the old kingdom, about 2800 B.C.

Heroditus

Slide 2-4: Heroditus

Encarta ‘96 Encyclopedia. Funk and Wagnalls, 1995.

Heroditus (484?-425 BC), called the *Father of History*, was the first to write about the pyramids around 440 B.C.

In his *History* Heroditus says that the pyramids, already ancient, were covered with a mantle of highly polished stones joined with the greatest exactness.

Secrets of the Great Pyramid

The pyramids are claimed to have many "secrets;" that they are models of the earth, that they form part of an enormous star chart, that their shafts are aligned with certain stars, that they are part of par of a navigational system to help travelers in the desert find their way, and on and on.

In this unit we'll examine the claim that the Great Pyramid contains the Golden Ratio, whatever that is, and then look at the claim that the Great Pyramid squares the circle, whatever that is.

Golden Ratio

So what is this *Golden Ratio* that the Great Pyramid is supposed to contain?

A ***ratio*** is the quotient of two quantities. The ratio of a to b is a/b

The price/earnings ratio is the price of a share of stock divided by the earnings of that share.

Price/ Earnings

A ***proportion*** results when two ratios are set equal to each other. Thus if the ratio of a to b equals the ratio of c to d, we have the proportion,

a/b = c/d

Systems of Proportions

Throughout much of art history, artists and architects were concerned with the proportions of the parts of their works. For example, if you were designing a temple, you might want to make the ratio of its height any old number, or perhaps, for some reason, **a particular value.** In fact, we'll see that there were not only particular ratios that were preferred, but sometimes entire **systems of proportions**.

Some systems of proportions were based on:

1. The musical intervals

2. The Human Body

3. The Golden Ratio

We'll see as we go along that these systems of proportions will be recurring themes thoughout the course.

Definition of the Golden Ratio

The golden ratio is also called ***extreme and mean ratio***. According to Euclid,

A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the less.

Derivation of the Golden Ratio

Let smaller part = 1, larger part = Φ. Thus $\Phi$is the golden ratio. It is often designated by the greek letter phi, $\Phi$for Phideas, (fl. c. 490-430 BC), Athenian sculptor and artistic director of the construction of the Parthenon, who supposedly used the golden ratio in his work.

Then by the definition of the golden ratio,

$\Phi/ 1 = (1 + \Phi) / \Phi$

so

$$\Phi^2 = 1^2 + 1\Phi$$

and we get the quadratic equation,

$$\Phi^2 - \Phi - 1 = 0$$

As a project, solve this quadratic equation for the golden ratio Φ. You should get,

$$\Phi = 1/2 + \sqrt{5} / 2 \approx \mathbf{1.618}$$

Project: Do this derivation.

Geometric Construction of the Golden Ratio

Subdivide a square of side 1 into two equal rectangles. Then lay out a distance equal to the diagonal of one of these half-squares, plus half the side of the original square. The ratio of this new distance to the original side, 1, is the golden ratio.

Project: Do this construction for the golden ratio.

Project: Mathematically show that this construction gives the golden ratio.

Egyptian Triangle

Let's now return to the pyramids. If we take a cross-section through a pyramid we get a *triangle.* If the pyramid is the Great Pyramid, we get the so-called *Egyptian Triangle.* It is also called the *Triangle of Price, and the Kepler triangle*.

This triangle is special because it supposedly contains the golden ratio. In particular,

the ratio of the slant height s to half the base b is said to be the golden ratio.

To verify this we have to find the slant height.

Computation of Slant Height s

The dimensions, to the nearest tenth of a meter, of the Great Pyramid of Cheops, determined by various expeditions.

height = 146.515 m, and base = 230.363 m

Half the base is

230.363 ÷ 2 = 115.182 m

So,

$s^2 = 146.515 + 115.182^2 = 34{,}733\ m^2$

s = 18636.9 mm

Does the Great Pyramid contain the Golden Ratio?

Dividing slant height s by half base gives

186.369 ÷ 115.182 = 1.61804

which differs from Φ(1.61803) by only one unit in the fifth decimal place.

The Egyptian triangle thus has a base of 1 and a hypotenuse equal to Φ. Its height h, by the Pythagorean theorem, is given by

$h^2 = \Phi^2 - 1^2$

Solving for h we get a value of $\sqrt{\Phi}$.

Project: Compute the value for the height of the Egyptian triangle to verify that it is $\sqrt{\Phi}$.

Thus the sides of the Egyptian triangle are in the ratio

$$1 : \sqrt{\phi} : \phi$$

Kepler Triangle

The astronomer Johannes Kepler (1571-1630) was very interested in the golden ratio. He wrote, *"Geometry has two great treasures: one is the theorem of Pythagoras, the other the division of a line into mean and extreme ratios, that is Φ, the Golden Mean. The first way may be compared to a measure of gold, the second to a precious jewel."*

In a letter to a former professor he states the theorem, which I rephrase as:

If the sides of a right triangle are in geometric ratio, then the sides are

$$1 : \sqrt{\phi} : \phi$$

We recognize this as the sides of the Egyptian triangle, which is why its also called the *Kepler triangle.*

Project: Prove that If the sides of a right triangle are in geometric ratio, then the sides are

$1 : \sqrt{\phi} : \phi$

The Star Cheops

A British railway engineer, Robert Ballard, saw the pyramids on his way to Australia to become chief engineer of the Australian railways. He watched from a moving train how the relative appearance of the three pyramids on the Giza plateau changed. He concluded that they were used as sighting devices, and wrote a book with the grand title of *The Solution of the Pyramid Problem* in 1882.

He also noted that the cross-section of the Great Pyramid is two of what we have called Egyptian triangles. He then constructs what he called a *Star Cheops*, which, he says, "... *is the geometric emblem of extreme and mean ratio and the symbol of the Egyptian Pyramid Cheops."*

To draw a star Cheops:

- Draw vertical and horizontal axes.
- Using their intersection as center, draw two circles, radius 1, and radius $1 + \Phi$.
- Superscribe a square about the smaller circle. This will be the base of the pyramid,
- From the point where an axis cuts the outer circle, draw two lines to the corners of the square. The triangle obtained will be one face of the pyramid.
- Repeat the preceding step for the remaining three faces, getting a four-pointed star. Cut it out.
- Fold each triangular face up from the base forming the pyramid.

Project Draw a star Cheops. Fold it to quickly make a model pyramid.

Squaring the Circle

Slide 2-3: The Great Pyramid

National Geographic. April '88

Now we'll look at his other claim, that the Great Pyramid's dimensions also show **squaring of the circle**. But just what is that?

The problem of squaring the circle is one of constructing, using only compass and straightedge;

(a) a square whose perimeter is exactly equal to the perimeter of a given circle, or

(b) a square whose area is exactly equal to the area of a given circle.

There were many attempts to square the circle over the centuries, and many approximate solutions, some of which we'll cover. However it was proved in the ninteenth century that an exact solution was impossible.

Squaring of the Circle in the Great Pyramid

The claim is:

The perimeter of the base of the Great Pyramid equals the circumference of a circle whose radius equal to the height of the pyramid.

Does it? Recall from the last unit that if we let the base of the Great pyramid be 2 units in length, then

pyramid height = $\sqrt{\Phi}$

So:

Perimeter of base = 4 x 2 = **8 units**

Then for a circle with radius equal to pyramid height $\sqrt{\Phi}$.

Circumference of circle = $2\pi\sqrt{\Phi} \approx$ **7.992**

So the perimeter of the square and the circumference of the circle agree to less than 0.1%.

An Approximate Value for π in Terms of Φ

Since the circumference of the circle ($2\pi\sqrt{\Phi}$) nearly equals the perimeter of the square (8)

$$2\pi\sqrt{\Phi} \approx 8$$

we can get an approximate value for π,

$\pi \approx \mathbf{4} / \sqrt{\Phi} = 3.1446$

which agrees with the true value to better than 0.1%.

Area Squaring of the Circle

The claim here is:

The area of that same circle, with radius equal to the pyramid height equals that of a rectangle whose length is twice the pyramid height($\sqrt{\Phi}$) *and whose width is the width* (2) *of the pyramid.*

Area of rectangle = $2\,(\sqrt{\Phi})\,(\,2\,) = \mathbf{5.088}$

Area of circle of radius $\sqrt{\Phi} = \pi r^2 \approx \pi(\sqrt{\Phi})^2 \approx \pi\,\Phi = \mathbf{5.083}$

an agreement withing 0.1%

The Pizza-Cutter Theory

Suppose that the Egyptians didn't know anything about but laid out the pyramid using a measuring wheel, such as those used today to measure distances along the ground.

Take a wheel of any diameter and lay out a square base one revolution on a side. Then make the pyramid height equal to two diameters

By this simple means *you get a pyramid having the exact shape of the Great Pyramid containing perimeter-squaring of the circle and area squaring of the circle and, for no extra cost, the golden ratio!*

Project: Use a pizza cutter or a similar disk to construct a pyramid similar to the Great Pyramid.

Project: Show, by calculation, that using a measuring wheel as described will give a pyramid of the same shape as the Great Pyramid.

Project: Find the diameter of the measuring wheel required so that:

100 revolutions = the base of the Great Pyramid

200 diameters = the height of the Great Pyramid

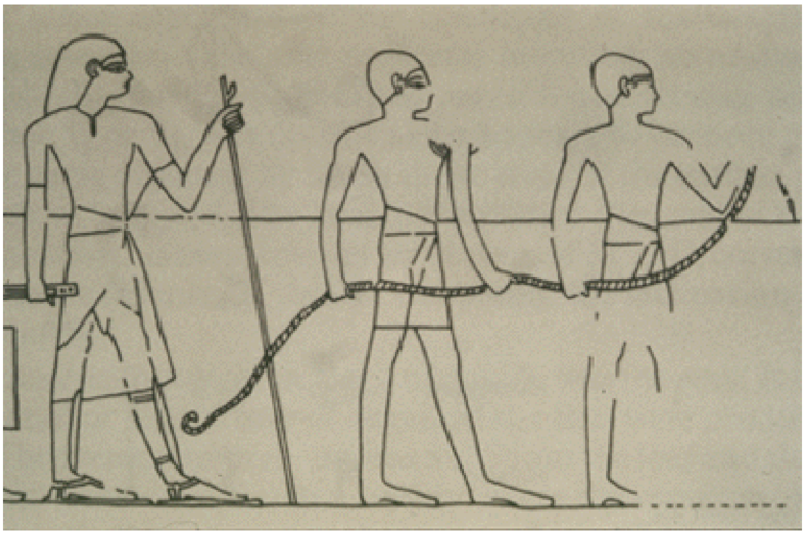
We'll see that this idea of squaring the circle will be a recurring theme throughout most of this course. But lets leave it for now and get back to triangles.

Rope-Stretcher's Triangle

One practical value of any triangle is its **rigidity**. A triangular frame is rigid, while a four-sided one will collapse.

Another imortant use is for **triangulation**, for locating things as in surveying and navigation, and this property takes us back to the very origins of geometry, in ancient Egypt.

The Origins of Geometry



Slide 2-5: Hardenonaptai: Rope stretchers or engineers

Tompkins, Peter. *Secrets of the Great Pyramid.* NY: Harper, 1971. p. 22

Geometry means *earth measure*. Geo + Metry. According to the Heroditus the Nile flooded its banks each year, obliterating the markings for fields.

He wrote, "*This king divided the land . . . so as to give each one a quadrangle of equal size and . . . on each imposing a tax. But everyone from whose part the river tore anything away . . . he sent overseers to measure out how much the land had become smaller, in order that the owner might pay on what was left . . . In this way, it appears to me, geometry originated, which passed thence to Greece.*

The Rope-stretcher's Triangle

One tool they may have used is a rope knotted into 12 sections stretched out to form a 3-4-5 triangle. Does it Produce a Right Angle?

According to the Pythagorean theorem,

In a right triangle, the square of the hypotenuse equals the sum of the squares of the legs.

The converse of is also true,

If the square of one side of a triangle equals the sum of the squares of the other two sides, then we have a right triangle.

For the 3-4-5 triangle;

$5^2 = 3^2 + 4^2$

$25 = 9 + 16$

It checks, showing a rope knotted like this will give a right angle.

The rope-stretcher's triangle is also called the 3-4-5 right triangle, the Rope-Knotter's triangle, and the Pythagorean triangle.

Project: Use a long knotted rope to make a rope-stretcher's triangle. Use it outdoors to lay out a right angle on some field. Then continue, making three more right angles to form a square. How accurate is your work? Did you come back to the starting point?

Summary

Was the golden ratio intentionally built into the Great Pyramid of Cheops? Why would anyone intentionally build the golden ratio into a pyramid, or other structure? What was the significance of to the Egyptians? And did the ancient Egyptians intentionally design the Great Pyramid to square the circle?

Its hard to know, but at any rate, we've introduced the golden ratio and squaring the circle themes which we'll encounter many times again in this study.

We've also some symbolism here:

If flooding of the Nile symbolized the annual return of watery chaos, then geometry, used to reestablish the boundaries, was perhaps seen as restoring law and order on earth. We'll see this notion again of geometry being sacred because it represents order, especially in the Middle Ages.

The rope stretchers triangle when opened out gives a zodiac circle, with the number of knots the most important of the astrological numbers

The square, with its four corners like the corners of a house, represents earthly things, while the circle, perfect, endless, infinite, has often been taken to represent the divine or godly. So squaring the circle is a universal symbol of bringing the earthly and mundane into a proper relationship with the divine.

And the Golden Ratio reverberates with the idea of the Golden Mean, the principle of moderation, defined by Aristotle as the mean between the two extremes of excess and insufficiency, as generosity is the mean between prodigality and stinginess, and by Horace, called the philosopher of the golden mean, advocated moderation even in the pursuit of virtue.

Remember that the pyramids were tombs, and that much of Egyptian art is funarary art. One Egyptian word for *sculptor* literally means *He who keeps alive*. To help the king acheive immortality, it was important that he didn't rot, hence the elaborate embalming. But embalming was not enough. The likeness of the king must also be preserved in gold or granite. So the tomb was seen as a sort of *life insurance policy*. Thus sculpture evolved.

But there is another angle to sculptor ... he who keeps alive. Once, the servants and slaves were buried with the king to help him in the other world. Then art came to the rescue, providing carved and painted substitutes for the real people. So the sculptor not only kept alive the memory of the dead king but literally kept alive all these people that would have been buried with the king.

Who says art isn't important?

Slide 2-7: King Tutankhamun

Metropolitan Museum of Art Gift Catalog, The Treasures of Tutahkhamun. NY: Met 1978.

Finally, in these units on Egypt we've started down the road that we'll follow right to the present time. The art historian Ernst Gombrich writes,

". . . the story of art as a continuous effort does not begin in the caves of southern France or among the North American Indians. . . there is no direct tradition which links these strange

beginnings with our own days . . .But there ***is*** *a direct tradition, handed down from master to pupil . . . which links the art of our own days with the art of the Nile valley some 5000 years ago. . ."*.. the Greek masters went to school with the Egyptians, and we are all the pupils of the Greeks.

In our next unit we'll cross the Mediteranean Sea where we too will be pupils of the Greeks.

Projects

Do the derivation of the golden ratio.

Do the construction for the golden ratio.

Compute the value for the height of the Egyptian triangle to verify that it is $\sqrt{\Phi}$.

Prove that If the sides of a right triangle are in geometric ratio, then the sides are $1 : \sqrt{\phi} : \phi$.

Draw a star Cheops. Fold it to quickly make a model pyramid.

Use a pizza cutter or a similar disk to construct a pyramid similar to the Great Pyramid.

Show, by calculation, that using a measuring wheel as described will give a pyramid of the same shape as the Great Pyramid.

Find the diameter of the measuring wheel required so that:

100 revolutions = the base of the Great Pyramid
200 diameters = the height of the Great Pyramid

Use a long knotted rope to make a rope-stretcher's triangle. Use it outdoors to lay out a right angle on some field. Then continue, making three more right angles to form a square. How accurate is your work? Did you come back to the starting point?

Reading

Markowsky, *Misconceptions Concerning the Golden Ratio.*

Tompkins, Chapter 16

Heroditus Book II, Paragraphs 124, 135

Euclid, *Elements.* P. 1, 2, Book 6, Definition 3.

Calter, pp. 156-171, pp. 548-551

Pythagoras
&
Music of the Spheres

There is geometry in the humming of the strings
... there is music in the spacing of the spheres.

Pythagoras.

The History of Philosophy (c.1660)
by Thomas Stanley.

Pythagoras

From Egypt we move across the Mediterranean Sea to the Greek island of Samos, the birthplace of Pythagoras, whose ideas dominate most of the material in this course. We'll introduce Pythagoras and his secret society of the Pythagoreans.

We'll look at the Pythagoreans' ideas about numbers, as a prelude to our next unit on number symbolism. Finally, we'll introduce a new idea that will be recurring theme throughout this course, the **musical ratios**, which will reappear in discussions of the architecture of the Renaissance.

Our main link between Egypt and Greece seems to be **Thales** c 640-550 BC, father of Greek mathematics, astronomy, and Philosophy, and was one of the Seven Sages of Greece. A rich merchant, his duties as a merchant took him to Egypt, and so became one of the main sources of Egyptian mathematical information in Greece. It was Thales advised his student to visit Egypt, and that student was **Pythagoras.**

Raphael's *School of Athens*

Slide 3-1: Raphael's *School of Athens* 1510-11.

Janson, H. W. *History of Art.* Fifth Edition. NY: Abrams, 1995. p.497

Pythagoras is shown in this famous painting, done by Raphael in 1510-11, which also shows most of the Greek philosophers.

Socrates sprawls on the steps at their feet, the hemlock cup nearby.

His student **Plato** the idealist is on the left, pointing upwards to divine inspiration. He holds his *Timaeus,* a book we'll talk about soon.

Plato's student **Aristotle**, the man of good sense, stands next to him. He is holding his *Ethics* in one hand and holding out the other in a gesture of moderation, the **golden mean**.

Euclid is shown with compass, lower right. He is the Greek mathematician whose *Elements* we'll mention often.

Slide 3-2: Pythagoras in Raphael's *School of Athens*

Janson, H. W. *History of Art.* Fifth Edition. NY: Abrams, 1995. p.497

Finally, we see **Pythagoras** (582?-500? BC), Greek philosopher and mathematician, in the lower-left corner.

The Pythagoreans

Pythagoras was born in Ionia on the island of Sámos, and eventually settled in Crotone, a Dorian Greek colony in southern Italy, in 529 B.C.E. There he lectured in philosophy and mathematics.

He started an academy which gradually formed into a society or brotherhood called the *Order of the Pythagoreans.*

Disciplines of the Pythagoreans included:

silence	music	incenses	physical and moral purifications
rigid cleanliness	a mild ascetisicm	utter loyalty	common possessions
secrecy	daily self-examinations (whatever that means)		
pure linen clothes			

We see here the roots of later monastic orders.

For badges and symbols, the Pythagoreans had the *Sacred Tetractys* and the *Star Pentagram*, both of which we'll talk about later.

There were three degrees of membership:

1. novices or "Politics"
2. Nomothets, or first degree of initiation
3. Mathematicians

The Pythagoreans relied on oral teaching, perhaps due to their pledge of secrecy, but their ideas were eventually committed to writing. Pythagoras' philosophy is known only through the work of his disciples, and it's impossible to know how much of the "Pythagorean" discoveries were made by Pythagoras himself. It was the tradition of later Pythagoreans to ascribe everything to the Master himself.

Pythagorean Number Symbolism

The Pythagoreans *adored* numbers. Aristotle, in his *Metaphysica,* sums up the Pythagorean's attitude towards numbers.

"The (Pythagoreans were) ... the first to take up mathematics ... (and) thought its principles were the principles of all things. Since, of these principles, numbers ... are the first, ... in numbers they seemed to see many resemblances to things that exist ... more than [just] air, fire and earth and water, (but things such as) justice, soul, reason, opportunity ..."

The Pythagoreans knew just the positive whole numbers. Zero, negative numbers, and irrational numbers didn't exist in their system. Here are some Pythagorean ideas about numbers.

Masculine and Feminine Numbers

Odd numbers were considered masculine; even numbers feminine because they are weaker than the odd. When divided they have, unlike the odd, nothing in the center. Further, the odds are the master, because odd + even always give odd. And two evens can never produce an odd, while two odds produce an even.

Since the birth of a son was considered more fortunate than birth of a daughter, odd numbers became associated with good luck. *"The gods delight in odd numbers,"* wrote Virgil.

---

1 Monad. Point. The source of all numbers. Good, desirable, essential, indivisible.

2 Dyad. Line. Diversity, a loss of unity, the number of excess and defect. The first feminine number. Duality.

3 Triad. Plane. By virtue of the triad, unity and diversity of which it is composed are restored to harmony. The first odd, masculine number.

4 Tetrad. Solid. The first feminine square. Justice, steadfast and square. The number of the square, the elements, the seasons, ages of man, lunar phases, virtues.

5 Pentad. The masculine marriage number, uniting the first female number and the first male number by addition.

- The number of fingers or toes on each limb.
- The number of regular solids or polyhedra.

Incorruptible: Multiples of 5 end in 5.

6 The first feminine marriage number, uniting 2 and 3 by multiplication.
The first *perfect* number (One equal to the sum of its aliquot parts, IE, exact divisors or factors, except itself. Thus, (1 + 2 + 3 = 6).
The area of a 3-4-5 triangle

7 Heptad. The maiden goddess Athene, the virgin number, because 7 alone has neither factors or product. Also, a circle cannot be divided into seven parts by any known construction).

8 The first cube.

9 The first masculine square.
Incorruptible - however often multiplied, reproduces itself.

10 Decad. Number of fingers or toes.
Contains all the numbers, because after 10 the numbers merely repeat themselves.
The sum of the archetypal numbers (1 + 2 + 3 + 4 = 10)

27 The first masculine cube.

28 Astrologically significant as the lunar cycle.
It's the second perfect number (1 + 2 + 4 + 7 + 14 = 28).
It's also the sum of the first 7 numbers (1 + 2 + 3 + 4 + 5 + 6 + 7 = 28)!

35 Sum of the first feminine and masculine cubes (8+27)

36 Product of the first square numbers (4 x 9)
Sum of the first three cubes (1 + 8 + 27)
Sum of the first 8 numbers (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8)

Figured Numbers

The Pythagoreans represented numbers by patterns of dots, probably a result of arranging pebbles into patterns. The resulting *figures* have given us the present word *figures.*

Thus 9 pebbles can be arranged into 3 rows with 3 pebbles per row, forming a square.

Similarly, 10 pebbles can be arranged into four rows, containing 1, 2, 3, and 4 pebbles per row, forming a triangle.

From these they derived relationships between numbers. For example, noting that a square number can be subdivided by a diagonal line into two triangular numbers, we can say that a square number is always the sum of two triangular numbers.

Thus the square number 25 is the sum of the triangular number 10 and the triangular number 15.

Sacred Tetractys

One particular triangular number that they especially liked was the number ten. It was called a *Tetractys,* meaning a set of four things, a word attributed to the Greek Mathematician and astronomer Theon (c. 100 CE). The Pythagoreans identified ten such sets.

Ten Sets of Four Things

Numbers	**1**	**2**	**3**	**4**
Magnitudes	point	line	surface	solid
Elements	fire	air	water	earth
Figures	pyramid	octahedron	icosahedron	cube
Living Things	seed	growth in length	in breadth	in thickness
Societies	man	village	city	nation
Faculties	reason	knowledge	opinion	sensation
Seasons	spring	summer	autumn	winter

Ages of a Person	infancy	youth	adulthood	old age
Parts of living things	body		three parts of the soul	

Gnomons

Gnomon means *carpenter's square* in Greek. Its the name given to the upright stick on a sundial. For the Pythagoreans, the gnomons were the *odd integers*, the masculine numbers. Starting with the monad, a square number could be obtained by adding an L-shaped border, called a gnomon.

Thus, the sum of the monad and any consecutive number of gnomons is a square number.

$$1 + 3 = 4$$

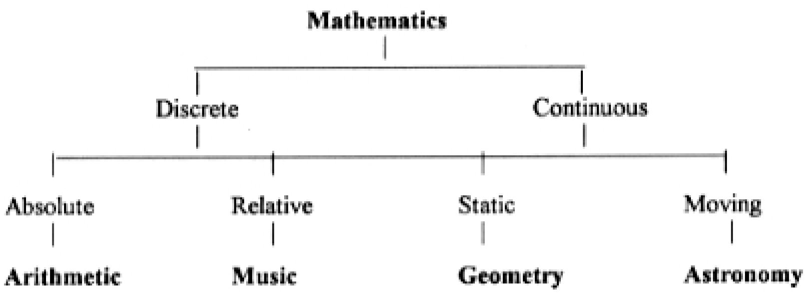
$$1 + 3 + 5 = 9$$

$$1 + 3 + 5 + 7 = 16$$

and so on.

The Quadrivium

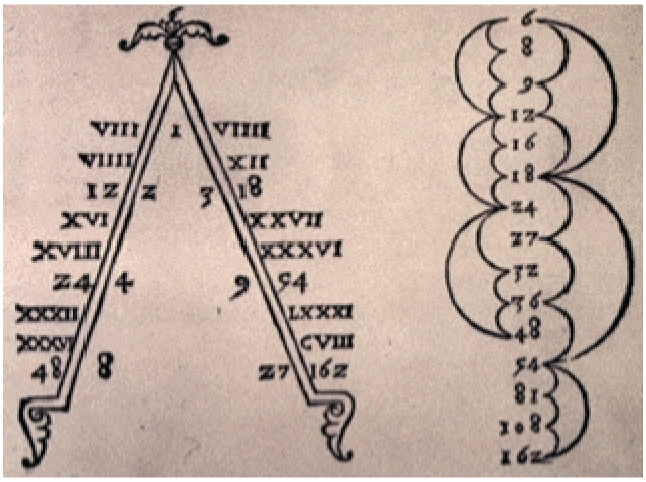
While speaking of groups of four, we owe another one to the Pythagoreans, the division of mathematics into four groups,



giving the famous ***Quadrivium of knowledge,*** the four subjects needed for a bachelor's degree in the Middle Ages.

Music of the Spheres

Jubal and Pythagoras



Slide 3-4: Theorica Musica
F. Gaffurio, Milan, 1492
Lawlor, Robert. *Sacred Geometry.* NY: Thames & Hudson, 1982. p.7

So the Pythagoreans in their love of numbers built up this elaborate number lore, but it may be that the numbers that impressed them most were those found in the **musical ratios.**

Lets start with this frontispiece from a 1492 book on music theory.

The upper left frame shows **Lubal** or **Jubal**, from the Old Testament, *"father of all who play the lyre and the pipe"* and 6 guys whacking on an anvil with hammers numbered 4, 6, 8, 9, 12, 16.

The frames in the upper right and lower left show Pithagoras hitting bells, plucking strings under different tensions, tapping glasses filled to different lengths with water, all marked 4, 6, 8, 9, 12, 16. In each frame he sounds the ones marked 8 and 16, an interval of 1:2 called the **octave**, or *diapason*.

In the lower right, he and Philolaos, another Pythagorean, blow pipes of lengths 8 and 16, again giving the octave, but Pythagoras holds pipes 9 and 12, giving the ratio 3:4, called the **fourth** or **diatesseron** while Philolaos holds 4 and 6, giving the ratio 2:3, called the **fifth** or **diapente**.

They are:

8 : 16 or	1 : 2	Octave	diapason
4 : 6 or	2 : 3	Fifth	diapente
9 : 12 or	3 : 4	Fourth	diatesseron

These were the only intervals considered harmonious by the Greeks. The Pythagoreans supposedly found them by experimenting with a single string with a moveable bridge, and found these pleasant intervals could be expressed as the **ratio of whole numbers**.

Pythagoras in the *School of Athens*

Slide 3-3: Closeup of Tablet

Bouleau

Janson, H. W. *History of Art.* Fifth Edition. NY: Abrams, 1995. p.497

Raphael's *School of Athens* shows Pythagoras is explaining the musical ratios to a pupil.

Notice the tablet. It shows:

The words ***diatessaron, diapente, diapason***.
The roman numerals for 6, 8, 9, and 12, showing the ratio of the intervals, same as in the music book frontispiece.

The word for the tone, **ΕΠΟΓΛΟΩΝ**, at the top.

Under the tablet is a **triangular number 10** called the **sacred tetractys**, that we mentioned earlier.

The Harmonic Scale

Slide 3-5: Gafurio Lecturing
F. Gafurio, *De Harmonia musicorum instrumentorum,* 1518, Wittkower, Rudolf. *Architectural Principles in the Age of Humanism.* NY: Random, 1965. 43a.

This diagram from a book written in 1518 shows the famous Renaissance musical theorist Franchino Gafurio with three organ pipes and 3 strings marked 3 , 4, 6. This indicates the octave, 3 : 6 divided by the harmonic mean 4, into the fourth, 3 : 4, and the fifth, 4 : 6 or 2 : 3.

The banner reads, "Harmonia est discordia concors" or *Harmony is discordant concord*, propounding the thesis that harmony results from two unequal intervals drawn from dissimilar proportions. The diagram shows compasses, suggesting a link between geometry and music.

So What?

So after experimenting with plucked strings the Pythagoreans discovered that the intervals that pleased people's ears were

octave	1 : 2
fifth	2 : 3
fourth	3 : 4

and we can add the two Greek composite consonances, not mentioned before . . .

octave plus fifth	1 : 2 : 3
double octave	1 : 2 : 4

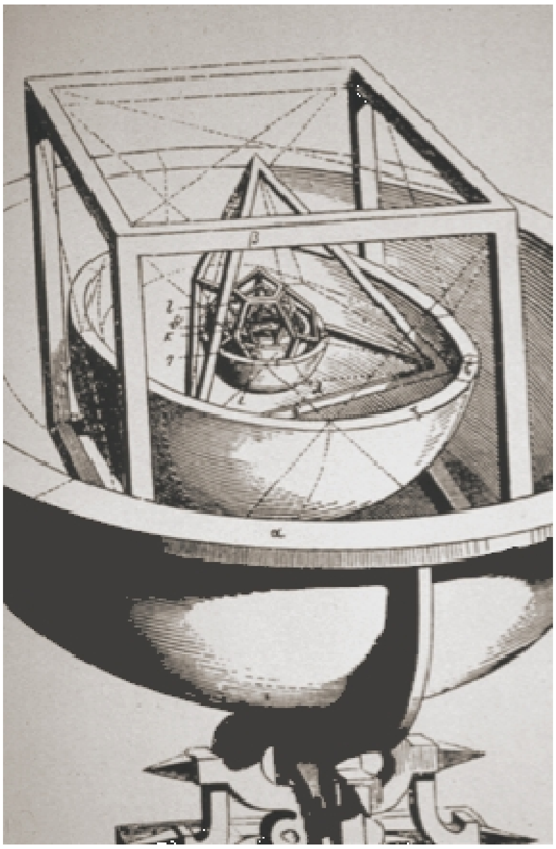
Now bear in mind that we're dealing with people that were so nuts about numbers that they made up little stories about them and arranged pebbles to make little pictures of them. Then they discovered that all the musical intervals they felt was beautiful, these five sets of ratios, were all contained in the simple numbers

1, 2, 3, 4

and that these were the very numbers in their beloved **sacred tetractys** that added up to the number of fingers. They must have felt they had discovered some basic laws of the universe.

Quoting Aristotle again ... *"[the Pythagoreans] saw that the ... ratios of musical scales were expressible in numbers [and that] .. all things seemed to be modeled on numbers, and numbers seemed to be the first things in the whole of nature, they supposed the elements of number to be the elements of all things, and the whole heaven to be a musical scale and a number."*

Music of the Spheres



Slide 3-6: Kepler's Model of the Universe
Lawlor, Robert. *Sacred Geometry.* NY: Thames & Hudson, 1982. p. 106

"... and the whole heaven to be a musical scale and a number... "

It seemed clear to the Pythagoreans that the distances between the planets would have the same ratios as produced harmonious sounds in a plucked string. To them, the solar system consisted of ten spheres revolving in circles about a central fire, each sphere giving off a sound the way a projectile makes a sound as it swished through the air; the closer spheres gave lower tones while the farther moved faster and gave higher pitched sounds. All combined into a beautiful harmony, ***the music of the spheres.***

This idea was picked up by Plato, who in his *Republic* says of the cosmos; ". . . *Upon each of its circles stood a siren who was carried round with its movements, uttering the concords of a* *single scale,"* and who, in his *Timaeus*, describes the circles of heaven subdivided according to the musical ratios.

Kepler, 20 centuries later, wrote in his *Harmonice Munde* (1619) says that he wishes *"to erect the magnificent edifice of the harmonic system of the musical scale . . . as God, the Creator Himself, has expressed it in harmonizing the heavenly motions."*

And later, *"I grant you that no sounds are given forth, but I affirm . . . that the movements of the planets are modulated according to harmonic proportions."*

Systems of Proportions based on the Musical Ratios

Slide 17-1: Villa Capra Rotunda
citatation

What does this have to do with art or architecture? The idea that the same ratios that are pleasing to the ear would also be pleasing to the eye appears in the writings of Plato, Plotinus, St. Augustine, and St. Aquinas. But the most direct statement comes from the renaissance architect Leone Battista Alberti (1404-1472), *"[I am] convinced of the truth of Pythagoras' saying, that Nature is sure to act consistently . . . I conclude that the same numbers by means of which the agreement of sounds affect our ears with delight are the very same which please our eyes and our minds."*

Alberti then gives a list of ratios permissible, which include those found by Pythagoras. We'll encounter Alberti again for he is a central figure in the development of perspective in painting.

We'll also discuss another architect who used musical ratios, Andrea Palladio (1518-1580), who designed the Villa Capra Rotunda shown here.

Summary

Slide 3-7: Correspondence School in Crotone

W. S. Anglin. *Mathematical Intelligence* V19, No. 1, 1997

I always wanted to make a pilgrimage to Crotone, site of the Pythagorean cult, but this is all that's there to mark their presence. Pythagoras and his followers died when their meetinghouse was torched. We'll have more on the Pythagoreans later, in particular their fondness for the star pentagram.

In this unit we've had some Pythagorean number lore and soon we'll add to it by talking about number symbolism in general, especially numbers in astrology and the Old Testament. Somewhere I had read that one answer to the question, *Why study history?* was *To keep Pythagoras alive!* I've forgotten where I read that, but anyway, it makes a nice goal for this course.

Reading

Newman, p. 78-89

Bell article, Art Bulletin December '95

Richter, p. 8-12

Clark, *Civilization*, p. 131-132

NUMBER SYMBOLISM

"But thou hast arranged all things by measure and number and weight."

Book of Wisdom of Solomon, from the OT Apocrypha., XI, 20.

Elementary Number Symbolism

The Pythagoreans were not the only ones who gave symbolic meaning to numbers. According to Vincent Hopper, *"nothing in the history of number symbolism is so striking as the unanimity of all ages and climates in regard to the meanings of a few number symbols."* He refers to this as *elementary number symbolism*.

In addition, we'll include numbers from mythology, astrology, and the Old Testament, which is loaded with symbolic numbers. We'll cover the New Testament and Christian number symbolism in a later unit.

We'll be able to connect some of the numbers used symbolically to larger ideas, but others will seem completely arbitrary. For those, our only justification in mentioning them is their repeated use as art motifs, and whose recognition may help us to understand a work of art.

Numero Uno

HILDEGARD: ***Man in Sapphire Blue***
Fox, p. 22

According to Hopper, the first advance towards counting is with the use of words for *one* and for *many,* the differentiation from the self from the group. We still say *numero uno* to speak of ourselves.

One - the first - the greatest - the beginning - is usually identified with the Creator. *In the beginning God created . . .*, so in the beginning there was only **one.**

Two & Duality

Slide 4-3: Janus
Cayley. *Classical Myths in English Literature.* Boston: Ginn 1893 p. 89

The number *two* appears to always have carried with it the idea of duality, of opposites and mutual antithesis, as we've seen with the Pythagoreans. Here is the Greek god Janus, for whom January is named. He sees both ways, both inwards and outwards, and has the wisdom of both past and future.

Slide 4-4: DÜRER: ***Adam and Eve***
Dürer, Albrecht. The Complete Engravings, Etchings and Drypoints. Ed. By Walter Strauss. NY: Dover, 1972.

Aristotle said, "*most human things go in pairs*." We have many sets of dualities, like night and day, hot, cold, etc. and especially male, female.

sun	moon
light	darkness
heat	cold
fire	water
day	night
active	passive
male	female

Yin and Yang

The Chinese believed that heaven and earth produced everything by the interaction of opposites called *Yang and Yin,* the masculine *Yang* and the feminine *Yin,* just as offspring are the result of the interaction of a male and a female parent.

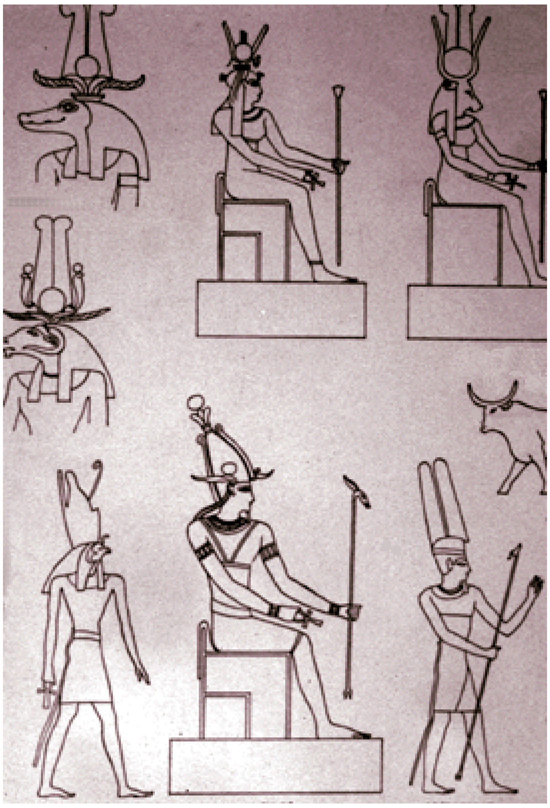
Famous Trios

Three represents the *triad* of family; male, female, and child; beginning, middle, and end; birth, life, and death.

Of two things we say *both;* of three things we say *all.*

Hopper writes that a single occurance is of no statistical significance, a second occupance might be just coincidence, but a third occurrence gives the event the impress of law, thus the Gypsy belief that if a dream comes three times it is certain to be true.

Egyptian Sun Gods



Slide 4-7: Egyptian Deities
New International Encyclopedia. NY: Dodd, 1917. Volume 7, p. 529

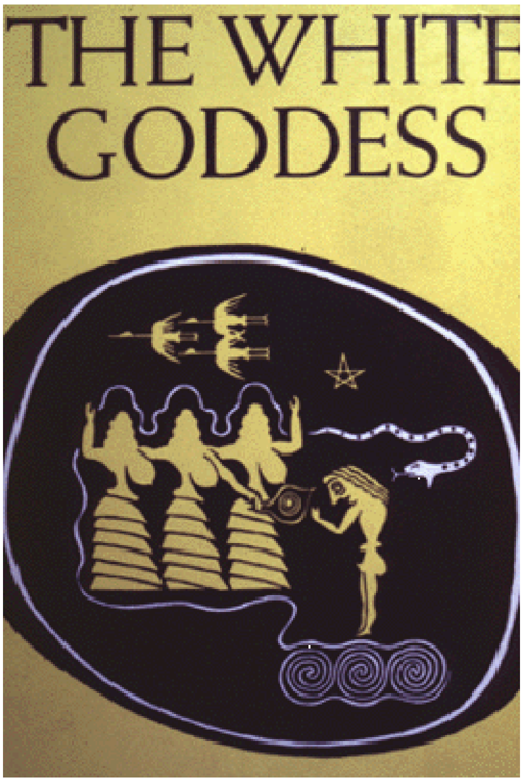
Three became the most universal number of deity, like the Holy Trinity that we'll cover in a later unit. There were three primary gods of Babylon *Anu, Bel, and Ea* representing heaven, earth, and the abyss.

Sun worship is one of the most primitive forms of religion, and early man sometimes distinguished between rising, midday, and setting sun. The Egyptians, for example, divided the sun god into three deities:

Horus, rising sun Ra or Rê, midday sun Osiris, old setting sun

---

Triple Goddess



Slide 4-8: White Goddess
Graves Cover

Just as three positions of the sun are represented by three deities, so with the moon.

Robert Graves writes about the *White Goddess*,

. . . the triple goddess of the New, Full, and Old Moon,

. . . goddess of Birth, Love, and Death . . .

. . . beautiful, generous, fickle, wise, implacable.

Three Fates

Slide 4-10: MICHELANGELO. *Three Fates*
Cayley. *Classical Myths in English Literature.* Boston: Ginn 1893 p. 72

There are lots of trios of women in mythology and in art . . .

Recall from our unit on *Music of the Spheres* that the circles of heaven turned and on each was a siren, each singing a single note. Plato, *Republic*, Chap XL, says that on the circles were the *three fates*, daughters of Necessity, the three sisters that controlled destiny, chanting to the Sirens' music. Lacheses of things past, Clotho of things present, Atropos of things to come, like the three ghosts in Dickens' *A Christmas Story.*

The length of someone's life was determined by the length of a single thread. Clotho held the spool, Lachesis pulled the thread, Atropos *snipped.*

Three Sirens

Plato's sirens are not to be confused with the three sirens, sea nymphs in the Mediterranean whose singing lead men to their deaths on the rocks. In the Odyssey, Odysseus resists them by tying himself to the mast and plugging the sailor's ears with wax. They symbolize the hazards faced by early mariners.

Three Witches

Slide 4-11: FUSELI: *Three Witches*
Clark, Kenneth, *The Romantic Rebellion.* NY. Harper, 1972.

Recall the famous lines from Macbeth,

Double, double, toil and trouble; Fire burn and cauldron bubble.

They appear to be making hot soup for lunch, and Shakespeare even gives the recipe, poison'd entrails, selter'd venom, fillet of fenny snake, eye of newt, toe of frog . . . and much more.

Three Furies

These three lovelies are sometimes called *Resentful, Relentless, and Avenger*. Their heads were covered with serpents and they breathed vengeance and pestilence. They were often found in the company of Mars, god of war.

Three Graces

Slide 4-13: RAPHAEL: *The Three Graces,* 1505-6.
Fisher, Sally. *The Square Halo.* NY: Abrams, 1995. p. 166

A more appealing trio of women is **The Three Graces**, sometimes called Splendor, Mirth, and Good Cheer, or Beauty, Gentleness, and Friendship. They were often shown with Venus, the nine Muses, and Apollo. The three graces is a very popular art motif, with involved iconography, but might have been just an excuse for artists to portray nude women.

Judgement of Paris

Slide 4-19: RUBENS: *Judgement of Paris*
American Library Color Slide Company
Slide # 863

A trio of women is also the subject of another favorite art motif, the *Judgement of Paris*. Paris was the son of the king and queen of Troy. They heard a prophecy that Paris would be the ruin of Troy so put him on Mount Ida where he was brought up by shepherds.

Years later the three goddesses, Hera, Athena, and Aphrodite, were at a wedding where someone tossed a golden apple into the crowd, inscribed *"To the Fairest."* All three wanted it and asked Zeus to decide, but he was too smart to get involved. He packed them all off to Mount Ida for Paris to make the decision.

Hera and Athena offered him riches, fame, empire, and military glory, but when Aphrodite bribed him with the offer of Helen, whom, he voted for Aphrodite, even though Helen was already married and Paris himself was in love with a nymph. What's more, Paris had to go and abduct Helen, which started the Trojan War.

This motif was also used as an allegory representing a choice between the active life and the sensuous one.

Earthly Fours

The number *four* is associated with the earth in many ways.

Four Ages of the World

Ovid writes of the Four Ages of the World:

Gold, the first, free of fear and conflict
Silver, the second, where man had to seek shelter
Bronze, the third, aggressive but not yet entirely evil
Iron, the fourth, with treachery, violence, greed deceit, and war.

Four Continents

Africa	Nile	crocodile, lion, snake, elephant
Americas	Plate	hunter with feathered headpiece
Asia	Ganges	camel, rhino, elephant
Europe	Nile	bull or horse

Slide 4-26: TITIAN: *Rape of Europa* c.1560.
Hartt, Frederic. *Italian Renaissance Art.* NY: Abrams, 1994. p. 609

Slide 4-27: GIORDANO: *Rape of Europa,* 1686
Janson, H. W. *History of Art.* Fifth Edition. NY: Abrams, 1995. p. 557

The Rape of Europa is a very popular art motif.

Zeus fell in love with the maiden Europa, disguised himself as a bull, fitting for this horniest of Gods, and abducted her to Crete. She eventually gave birth to the continent of Europe.

Four Cardinal Virtues

Slide 4-29: ANDREA DELLA ROBBIA: ***Allegory of Prudence***
Fisher, Sally. *The Square Halo.* NY: Abrams, 1995. c. 1475. p. 165

Plato, in his *Republic* mentions four virtues, *Prudence (or Wisdom), Fortitude (or Courage), Temperance, and Justice.* These are called the four cardinal virtues to distinguish them from the three Ecclesiastical virtues we'll see later in the New Testament.

Number Symbolism in the Old Testament

To elementary number symbolism we add the Old Testament, in which everything is numbered. Sometimes the numbers have symbolic meaning, but the numbers are often used, it seems, just to make the narrative more concrete, saying, for example, that *"Solomon built a house 100 cubits long"*instead of *"Solomon built a really long house."*

Umberto Eco says that much number symbolism can be attributed to the passage from the Book of Wisdom of Solomon, from the OT Apocrypha. XI, 20.

Four Cardinal Points

The Old Testament has many references to the four cardinal directions, such as in Isaiah 11, 12:

"And he shall ... gather together the dispersed of Judah from ***the four corners*** *of the earth."*

and Ezekiel 7, 2:

"An end, the end is come upon the ***four corners*** *of the land"*

Four Winds

Slide 4-25:
Kitzinger, Ernst. *The Art of Byzantium and the Medieval West.* Bloomington: Indiana, 1976. p. 333

The invention of the four winds is a simple enough addition. In Jeremiah 49, 36 we read

*"I bring **the four winds from the four quarters of heaven,** and will scatter them toward all those winds"*

From Virgil's *Aeneid* we have Aeolus, King of the Winds, who lived on an island just north of Sicily, who ruled the **four winds**:

Boreas, the north wind (Latin, Aquilo)
Zephyr, the west wing (Latin Favonius)
Notus, the south wind (Latin Auster)
Eurus, the east wind (same in Latin)

Four Rivers

Four rivers are mentioned in the Old Testament, Gen 2, 10:

" *And a river went out of Eden ... and parted ... into four heads.*
*The . . . first [is] **Pison** ... which compasses the whole land of Havilah ...*
*the second [is] **Gihon** ... that compasses the whole land of Ethiopia ...*
*the third [is] **Hiddekel ...** that goes toward the east of Assyria ...*
*and the fourth [is] **Euphrates** that goes eastward to Assyria.*"

Four Rivers of Hell

Slide 4-28: Aeneas and the Sibyl enter Charon's boat.
Hamilton, Edith. *Mythology.* NY: Mentor, 1942, p. 227

The **four rivers of Hades**, Acheron, Styx, Phlegethon, and Cocytus, are not from the Old Testament, but play an important role in Plato's *Phaedo*, Dante's *Inferno.* and Virgil's *Aeneid.* In this picture, Aeneas and the Sibyl enter Charon's boat to cross at the junction of the Acheron and the Cocytus.

Astrological Numbers

Slide 4-21: Phases of the Moon
"Maps of Heavens." The British Library

Another major source of number symbolism is Astrology, an ancient practice that seemed to develop independently in different civilizations.

The Chaldeans, who lived in Babylonia (now Iraq), developed astrology as early as 3000 B.C.E., and the Chinese were practicing astrology by 2000 BC. Astrology was known in ancient India and by the Maya of Central America. By the 500s BC, astrology had spread to Greece, where Pythagoras and Plato used it into their study of religion and astronomy.

It is certainly clear that certain astronomical bodies, particularly the sun, affected the change of seasons and the success of crops. So its not a big leap to assume that the movements of other bodies such as the planets affected or represented additional aspects of life. They also gave our notions of time, from the daily cycle of night and day, the lunar cycle of 28 days, giving the month, subdivided into the four phases of the moon, and the yearly cycle of 12 lunar cycles or about 360 daily cycles.

Four Seasons

Slide 4-22: Hades & Persephone
Hamilton, Edith. *Mythology.* NY: Mentor, 1942 p. 22

In Greek mythology, the seasons came about when Demeter's daughter, Persephone was abducted to the underworld by Hades. There followed an awful flap, too involved to relate here, but they finally cut a deal where Persephone spent half the year in the underworld, which time became autumn and winter, and half above ground, which became spring and summer.

Four Divisions of the Day

Slide 4-23: MICHELANGELO: Sculptures in Medici Chapel, closeup of Notte
Canaday, John. *Masterpieces by Michelangelo.* NY: Crown, 1979. p. 95

The division of the day into **4 periods**, dawn, day, evening, and night, is here represented by Michelangelo, in the Medici Chapel.

Four Cardinal Points

Slide 4-24: Compass Rose
Calter Photo

The movements of celestial bodies may also have led to Four becoming identified with the four directions, toward the sunrise, towards the sunset, and the two directions perpendicular to these.

Thus four is the number of the *cardinal points*, N, S, E, W, and the directions in which a person can move; right, left, forward, back.

Celestial Sevens

Slide 4-35: VEDDER: The Pleides
Cayley. *Classical Myths in English Literature.* Boston: Ginn 1893 p. 146

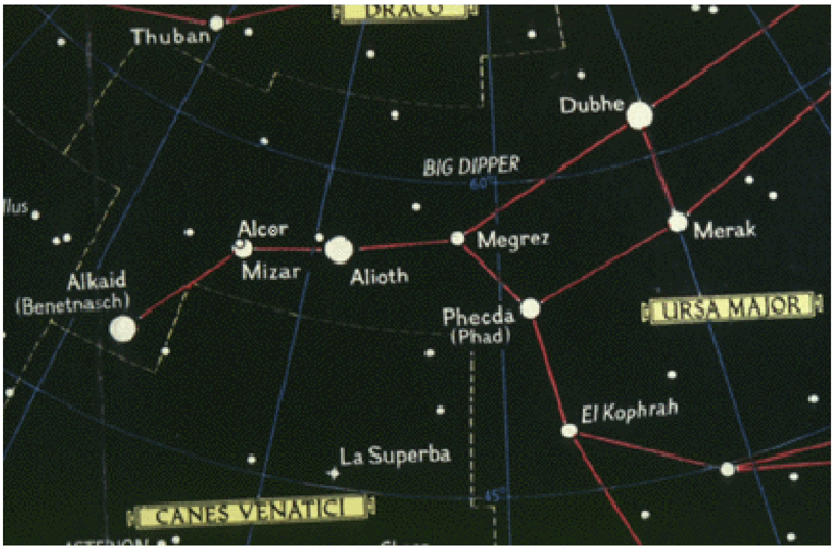
Seven is one of the main astrological numbers.

- Since 4 lunar phases made a complete lunar cycle of 28 days, each phase was 7 days, which defines the week.

- There are seven stars in the Pleides.

Many a night I saw the Pleides, rising thro' the mellow shade, Glitter like a swarm of fireflies tangled in a silver braid.

From *Locksley Hall* by Alfred, Lord Tennyson (1809-92)



Slide 4-36: Big Dipper
National Geographic Map

- There are seven stars in the Big Dipper, the most prominent of the "indestructible" stars, the circumpolar.

From Homer's *Iliad* we have; *". . . and the bear, which revolves in its place and watches Orion, and alone of [the stars] never takes a bath in the ocean."*

- There are seven colors of the rainbow: red, orange, yellow, green, blue, indigo, violet

We'll have more about the rainbow as an art motif later.

Planets

The ancient world knew of seven "planets" and identified them with the days of the week.

Day	**Planet**
Sunday	Sun
Monday	Moon
Tuesday	Mars
Wednesday	Mercury
Thursday	Jupiter
Friday	Venus
Saturday	Saturn

Wisdom

Seven is often used as a number of wisdom. From the Old Testament, Proverbs 9:1:

"Wisdom hath builded her house, she hath hewn out her seven pillars"

The Flood

Slide 4-37: DORE: *The Deluge*
O. M. Dunham. *"Bible Gallery,"* 1880

Other Sevens from the Old Testament include the great flood. **Noah** had seven days to prepare before the flood. He was commanded to take seven pair of clean beasts and birds.

*"Of every clean beast thou shalt take to thee **by sevens**, the male and the female: and of beasts that are not clean **by two**, the male and the female."*

Clean and unclean spelled out in Leviticus Chapter 11, where clean meant ritually pure, similar to what we consider Kosher.

The Creation

Slide 4-38:
MICHELANGELO:
Creation of the Sun, Moon, and Vegetation
Canaday, John. *Masterpieces by Michelangelo.* NY: Crown, 1979. p. 17

Other Sevens from the Old Testament include the Seven acts of creation;

1. light, 2. firmament, 3. plants, 4. heavenly bodies, 5. fish & fowl, 6. animals & humans, 7. rest.

Sabbatical & Jubilee

The notion that the seventh day was for rest led to the adoption of the weekly Sabbath as a day of rest. But other sacred times of the calendar were based on seven, the seventh or sabbatical year.

Leviticus 25: 1-4: *And . . . then shall the land keep a Sabbath . . . Six years shall you sow your field, and prune thy vineyard, and gather fruit. But the seventh year shall be a Sabbath of rest unto the land you shall neither sow your field, nor prune your vineyard.*

and the Jubilee year.

Leviticus 25, 8- 11: *And you shall number seven Sabbaths of years unto thee, seven times seven years; Then shalt thou cause the trumpet of the jubilee to sound . . . And ye shall hallow the fiftieth year, and proclaim liberty throughout the land it shall be a jubile unto you; and ye shall return every man unto his possession, and ye shall return every man unto his family. . . . ye shall not sow, neither reap that which groweth of itself in it, nor gather [the grapes]. . .*

Menorah

Slide 4-39: Menorah
Keller, Sharon. *The Jews: A Treasury of Art and Literature.* NY: Levin Assoc. 1992.

Seven also numbers the days of Passover and the branches of the menorah

Exodus 25, 31-37: *"And thou shalt make a candlestick [of] pure gold . . .. . six branches shall come out of the sides of it; three branches . . . out of the one side, . . . and three branches out of the other side . . . Three bowls made like unto almonds . . . in one branch. . . and three bowls made like almonds in the other branch. . . . And you shall make the seven lamps thereof."*

Slide 4-40: Menorah, lit

Calter Photo

Robert Graves gives the menorah cosmic significance by comparing the seven flames to the **seven planets.** He cites Zechariah 4:1-10,

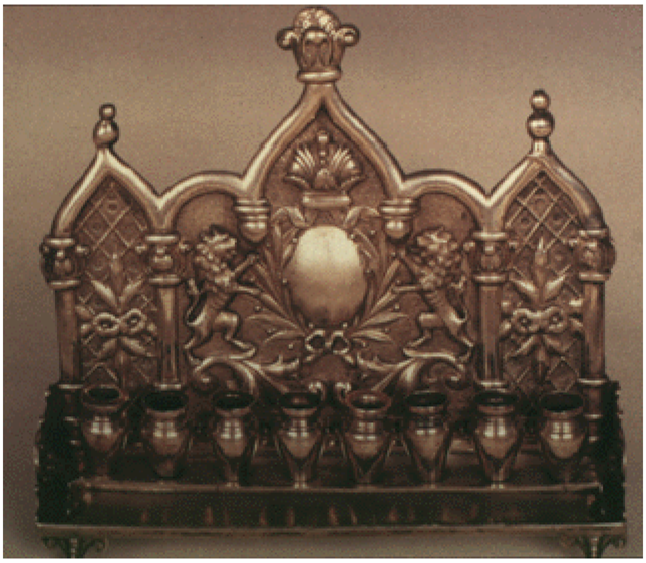
"And the angel . . . came again, and waked me, . . . And he said to me, What seest thou? And I said, I have seen, . . . a candlestick all of gold, with its bowl upon the top of it, and its seven

lamps thereon; . . . [these are] the eyes of Jehovah, which run to and fro through the whole earth."

Slide 4-41: Menorah, lit in a dark room
Calter Photo

Graves takes the *seven eyes* to mean the seven planets.

Chanukah Candlestick



Slide 4-43: Silver Menorah, Prague, 1860
Levin, Hugh, Associates. Jewish Calendar, 1997.

In contrast to the Menorah of Exodus with seven candles is the Chanukah menorah with eight candles, which celebrates a miracle. The story is that when the Temple in Jerusalem was rededicated after the war of the Macabees (167-160 B.C.E.) and after being desecrated by Antiochus Epiphanes, only a small amount of oil was found to light the menorah but it lasted for eight days.

A ninth candle, the Shamesh, is used to light the other eight, one night at a time, for the eight days of Chanukah, the festival of lights occurring near the winter solstice.

Ten Digits

Being the number of *fingers or toes,* ten became the base of the *decimal number system*. It is the number of *completeness or finality*.

With ten as *complete*, nine comes into prominence as *almost complete.* Troy was besieged for 9 days and fell on the tenth. Odysseus wandered for 9 years and arrived home on the tenth.

Ten Commandments

From the Old Testament we have, of course, the *ten commandments* or Decalogue. The Old Testament also describes ***Tithing,*** to give a tenth part of one's income.

Twelve and the Zodiac

Slide 4-48: Zodiac Pavement
Keller, Sharon. *The Jews: A Treasury of Art and Literature.* NY: Levin Assoc. 1992. p.20

Twelve is one of the big numbers in astrology because twelve complete lunar cycles takes approximately one year. Each month was eventually identified with a sign of the zodiac, which is believed to have originated in Mesopotamia as early as 2000 BC. The Greeks adopted the symbols from the Babylonians and passed them on to the other ancient civilizations.

The Egyptians assigned other names and symbols to the zodiacal divisions. The Chinese also adopted the 12-fold division, but called the signs rat, ox, tiger, hare, dragon, serpent, horse, sheep, monkey, hen, dog, and pig. The Aztecs independently devised a similar system.

Twelve thus symbolized a ***complete cycle***; and twelve was also used to divide the day and also the night.

Twelve Gods of Olympus

Slide 4-49: MANTEGNA: *Parnassus* American Library Color Slide Co. Inc. Slide #7507

Twelve is the number of Gods on Mount Olympus:

The Four siblings

1. Zeus	Jupiter	supreme ruler
2. Poseidon	Neptune	ruler of the sea
3. Hades	Pluto	ruler of the underworld
4. Hestia	Vesta	virgin. hearth & home
5. Hera	Juno	Zeus' wife

The Children of Zeus

6. Ares	Mars	war
7. Pallas Athena	Minerva	virgin

8. Hephaestus	Vulcan	fire, forge
9. Phoebus	Apollo	Apollo beautiful
10. Artemis	Diana	huntress, virgin
11. Hermes	Mercury	messenger
12. Aphrodite	Venus	love & beauty, mother of Aeneus

But, curiously, the twelve Olympians are not all associated with the 12 signs of the zodiac.

Twelve Labors of Hercules

Slide 4-50: POLLAIUOLO: *Hercules and Antaeus*
Janson, H. W. *History of Art.* Fifth Edition. NY: Abrams, 1995. p. 466

Twelve is associated with ***Hercules***, a very popular art motif. The Renaissance artist Pollaiuolo, for example, did an entire series of paintings on Hercules.

Slide 4-51: Herculaneum
Calter Photo

Hercules even had a city named for him, *Herculaneum*, destroyed in 79 AD by the same eruption of Vesuvius that buried Pompeii. The Italian city of *Ercolano* still bears his name.

Fatal Forties

Forty is the number of trial and privation, possibly starting when the Babylonians observed the forty-day disappearance of the Pleides which coincided with the rainy season, storms and floods, trial, danger. The Pleides' return marked the start of the New Year Festival. Other negative associations for forty are:

- 40 years of Hebrew wandering in the desert
- 40 days and nights of the great flood.
- 40 years of Philistine dominion over Israel
- 40 days of Moses on Sinai
- 40 days of Elijah's journey
- 40 days of mourning for Jacob
- 40 days, and Nineveh shall be overthrown, said Jonah.
- 40 days of Lent, the period of fasting, self-denial, and penitence traditionally observed by Christians in preparation for Easter.
- 40 day period of isolation in the Roman port, which survives in the word *quarantine.*

Summary

Tabulating some of our number symbolism, we have:

One	Self, Deity, Source of all Numbers
Two	Duality, Defect
Three	All, Best, Holy, First Triangular
Four	Astrological Number, Earth, First Square
Seven	Astrological, Steps to Wisdom and Virtue
Ten	Completeness, Finality, Perfection
Twelve	Zodiac, A Complete Cycle
Forty	Trial and Privation

We may laugh now at some of the meanings given to numbers, but how many of us would be willing to stay in room 13 or on the 13th floor on Friday the 13th?

We're not finished with number symbolism. We'll cover those associated with the New Testament, Christianity, and the Middle Ages later.

Keep in mind that we're on shaky ground here. Much of this material is speculation, without any real proof, and is full of contradictions.

So why did we do this? We're not so much interested in the numbers themselves but we are looking for frequently used art motifs. Further, seeing how the ancients viewed numbers will help us get into the right frame of mind to understand how they viewed geometric figures, triangles, squares, octagons, and that's the subject of our next unit.

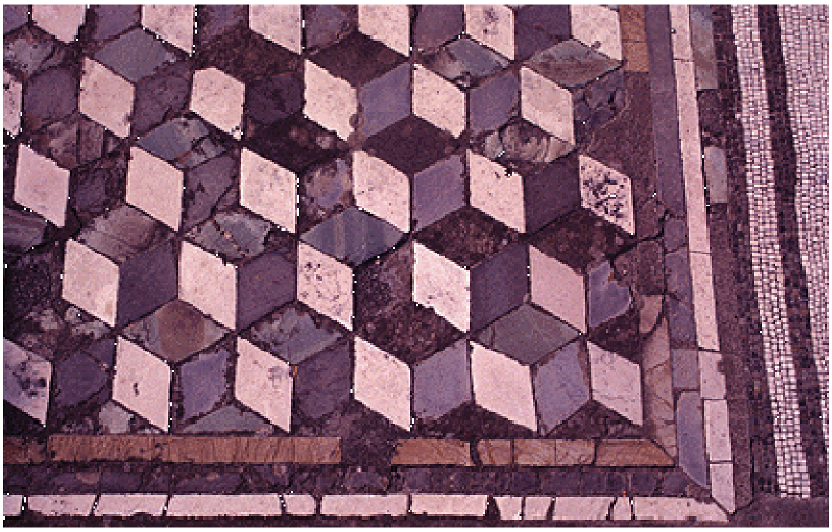
Reading

Butler, Chapter 1

Hopper, Chapters 1-3

Calter, pp. 1-3

Polygons, Tilings,
&
Sacred Geometry



Slide 5-1: Pompeii pavement
Calter photo

In the last unit, *Number Symbolism,* we saw that in the ancient world certain numbers had symbolic meaning, aside from their ordinary use for counting or calculating.

In this unit we'll show that the plane figures, the *polygons,* triangles, squares, hexagons, and so forth, were related to the numbers (three and the triangle, for example), were thought of in a similar way, and in fact, carried even more emotional baggage than the numbers themselves, because they were visual. This takes us into the realm of *Sacred Geometry.*

For now we'll do the polygons directly related to the Pythagoreans; the equilateral triangle (Sacred tetractys), hexagon, triangular numbers, and pentagram. We'll also introduce tilings, the art of covering a plane surface with polygons.

Polygons

Slide 5-23: Design at Pompeii

Calter photo

In the last unit, *Number Symbolism* we saw that in the ancient world certain numbers had symbolic meaning, aside from their ordinary use for counting or calculating. But each number can be associated with a plane figure, or *polygon* (Three and the Triangle, for example).

In this unit we'll see that each of these polygons also had symbolic meaning and appear in art motifs and architectural details, and some can be classified as *sacred geometry.*

A *polygon* is a plane figure bounded by straight lines, called the sides of the polygon.

From the Greek *poly* = many and *gon* = angle

The sides intersect at points called the *vertices.* The angle between two sides is called an *interior angle* or *vertex angle.*

Regular Polygons

A *regular polygon* is one in which all the sides and interior angles are equal.

Polygons vs. Poly*grams*

A poly*gram* can be drawn by connecting the vertices of a pol*gon.* Pentagon & Pentagram, hexagon & hexagram, octagon & octograms

---

Equilateral Triangle

Slide 5-2: Tablet in School of Athens, showing Tetractys

Bouleau

There are, of course, an infinite number of regular polygons, but we'll just discuss those with sides from three to eight. In this unit we'll cover just those with 3, 5, and 6 sides. We'll start with the simplest of all regular polygons, the equilateral triangle.

Sacred Tetractys

The Pythagoreans were particularly interested in this polygon because each triangular number forms an equilateral triangle. One special triangular number is the triangular number for what they called the *decad,* or *ten,* the **sacred tetractys.**

Ten is important because it is, of course, the number of fingers. The tetractys became a symbol of the Pythagorean brotherhood. We've seen it before in the *School of Athens.*

Trianglular Architectural Features

Slide 8-11: Church window in Quebec

In architecture, triangular windows are common in churches, perhaps representing the trinity.

Triskelion, Trefoil, Triquerta

Other three-branched or three-comered designs include the **triskelion**.

Slide 5-3: Greek Triskelion: Victory and Progress
Lehner, Ernst. *Symbols, Signs & Signets.* NY: Dover, 1950 p. 85

Slide 5-4: Irish Triskelions from Book of Durrow.
Met. Museum of Art. *Treasures of Early Irish Art.* NY: Met. 1977

Its a design that I liked so much I used it for one of my own pieces.

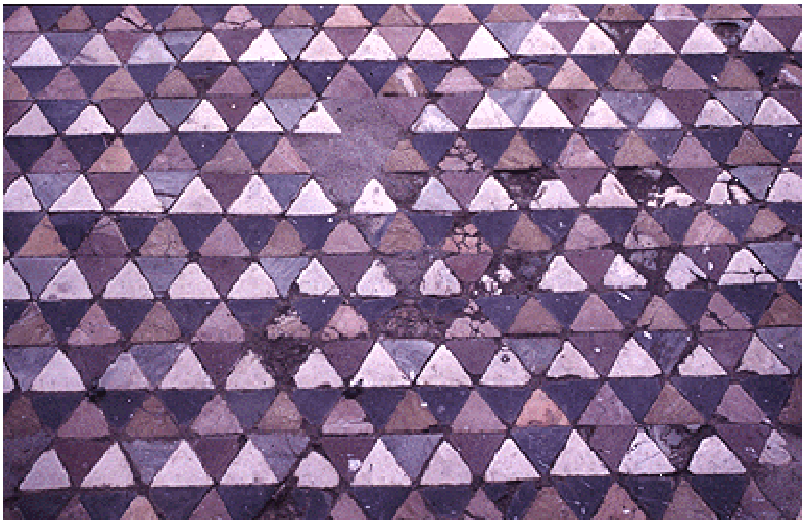
Slide 5-5: Calter carving Mandala II

Calter photo

Slide 5-6: Closeup of wheel

Calter photo

Tilings

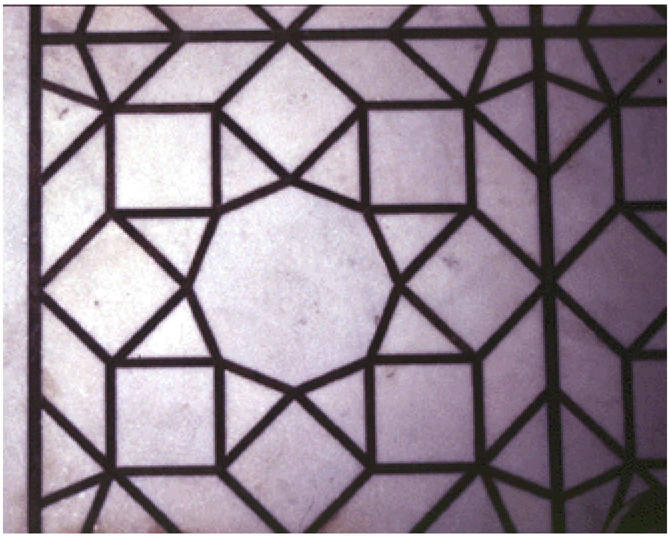


Slide 5-7: Pompeii Tiling with equilateral triangles

Calter photo

Tilings or *tesselations* refers to the complete covering of a plane surface by tiles. There are all sorts of tilings, some of which we'll cover later. For now, lets do the simplest kind, called a regular tiling, that is, tiling with *regular polygons*.

This is opposed to **semiregular tilings** like the Getty pavement shown here.

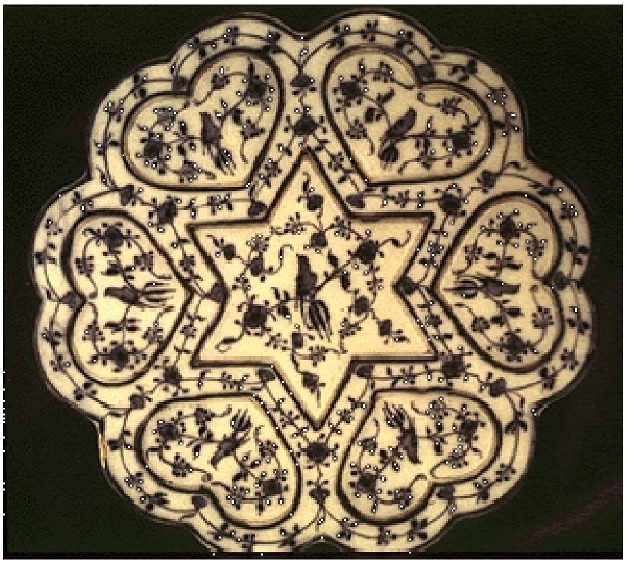


Slide 5-8: Getty Pavement

Calter photo

The equilateral triangle is one of the three regular polygons that tile a plane. the other two being the square and hexagon.

Hexagon & Hexagram

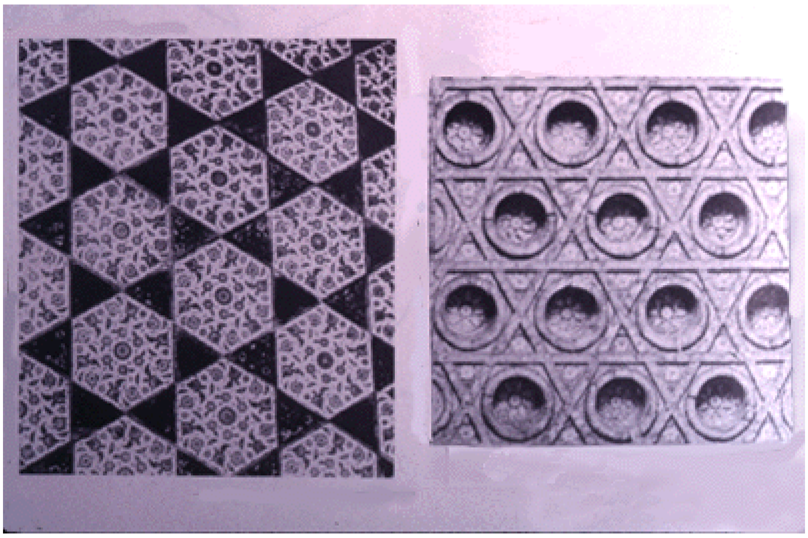


Slide 5-15: Plate with Star of David
Keller, Sharon. *The Jews: A Treasury of Art and Literature.* NY: Levin Assoc. 1992

Hexagonal Tilings

Our next polygon is the hexagon, closely related to the equilateral triangle

The hexagon is a favorite shape for tilings, as in these Islamic designs, which are not regular tilings, because they use more than one shape.

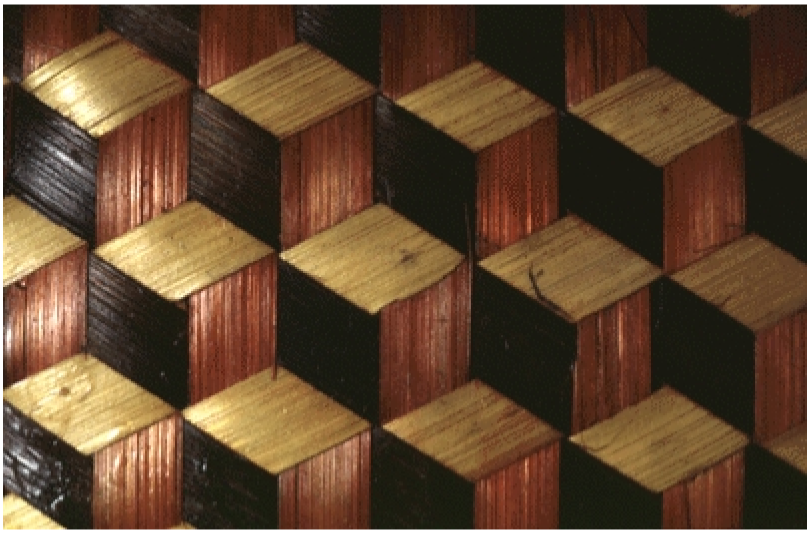


Slide 5-9: Islamic Tiling Patterns
El-Said, Issam, et al. *Geometric Concepts in Islamic Art.* Palo Alto: Seymour, 1976. p. 54

But, as we saw, the hexagon is one of the three regular polygons will make a regular tiling.

An Illusion

The hexagon is sometimes used to create the illusion of a cube by connecting every other vertex to the center, forming three diamonds, and shading each diamond differently.



Slide 5-10: Basket

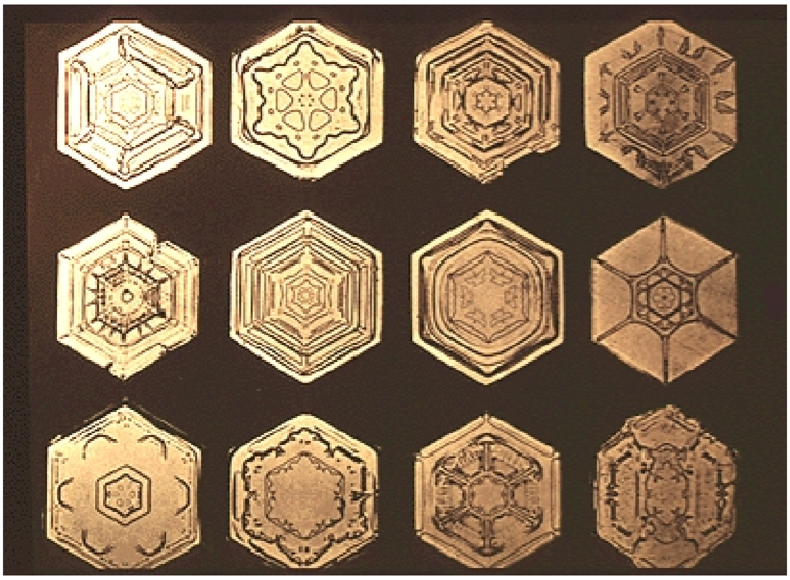
Calter photo

Slide 5-11: Pavement, Ducal Palace, Mantua

Calter photo

The Hexagon in Nature

The hexagon is found in nature in the honeycomb, and some crystals such as basalt, and of course, in snowflakes.



Slide 5-12: Snowflakes

Bentley, W. A. *Snow Crystals.* NY: Dover, 1962.

Six-Petalled Rose

The hexagon is popular in architectural decoration partly because it is so easy to draw. In fact, these are *rusty-compass constructions,* which could have been made with a forked stick.

Six circles will fit around a seventh, of the same diameter, dividing the circumference into 6 equal parts, and the radius of a circle exactly divides the circumference into six parts, giving a *six petalled rose.*

Slide 5-13: Moses Cupola. S. Marco, Venice
Demus, Otto. *The Mosaic Decoration of San Marco, Venice.* Chicago: U. Chicago, 1988. plate 60.

Hexagon vs. Hexagram

Connecting alternate points of a hexagon gives a hexagram, a six-pointed star, usually called the ***Star of David,*** found in the flag of Israel.

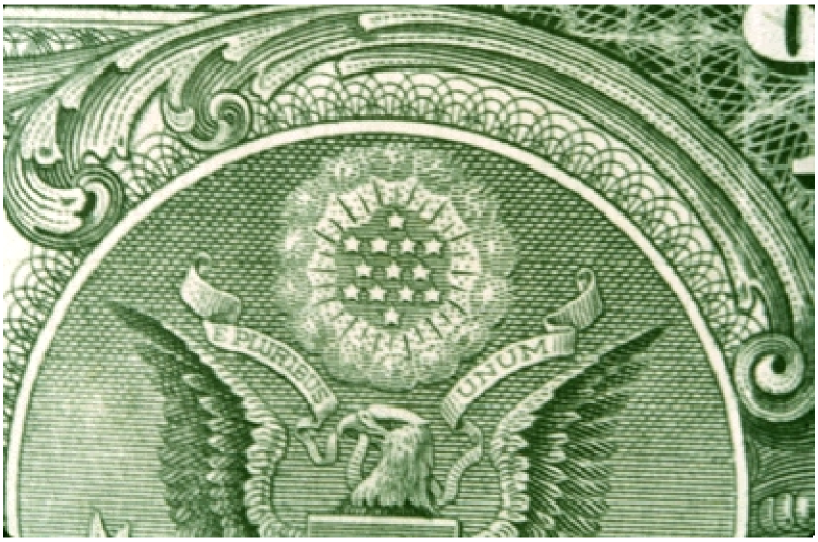
Slide 5-14: Star of David on Silver bowl from Damascus.
Jewish Museum (New York, N.Y.), Treasures of the Jewish Museum. NY: Universe, 1986. p. 61

Solomon's Seal

The hexagrarn is also called a *Solomon's Seal.* Joseph Campbell says that King Solomon used this seal to imprison monsters & giants into jars.

Slide 5-17: The genii emerging.
Burton, Richard. *The Arabian nights entertainments.* Ipswich : Limited Editions Club, 1954.

The U.S. Great Seal



Slide 5-20: Seal on Dollar Bill

Calter photo

The hexagrarn can also be viewed as two overlapping Pythagorean tetractys.

Joseph Campbell writes; *In the Great Seal of the U.S. there are two of these interlocking triangles. We have thirteen points, for our original thirteen states, and six apexes: one above, one below, andfour to thefour quarters. The sense of this might be thalftom above or below, orftom any point of the compass, the creative word may be heard, which is the great thesis of democracy.*

- The Power of Myth. p.27

Hexagonal Designs in Architecture

Hexagonal designs are common in ancient architecture, such as this church window in Quebec.

Slide 5-22: Church Window in Quebec

Calter photo

This marvelous design is at Pompeii. It is made up of a central hexagon surrounded by squares, equilateral triangles, and rhombi.

Slide: 5-23. Design at Pompeii
Calter photo

Slide 5-24: Design on Pisa Duomo
Calter photo

This hexagram is one of countless designs on the Duomo in Pisa.

Pentagon & Pentagram

Slide 5-26: Pentagram from grave marker

Calter photo

The Pentagram was used as used as a sign of salutaton by the Pythagoreans, its construction supposed to have been a jealously guarded secret. Hippocrates of Chios is reported to have been kicked out of the group for having divulged the construction of the pentagram.

The pentagram is also called the **Pentalpha,** for it can be thought of as constructed of five A's.

Euclid's Constructions of the Pentagon

Euclid gives two constructions in Book IV, as Propositions 11 & 12. According to the translator T.L. Heath, these methods were probably developed by the Pythagoreans.

Medieval Method of Construction

Supposedly this construction was one of the secrets of Medieval Mason's guilds. It can be found in Bouleau p. 64.

Durer's Construction of the Pentagon

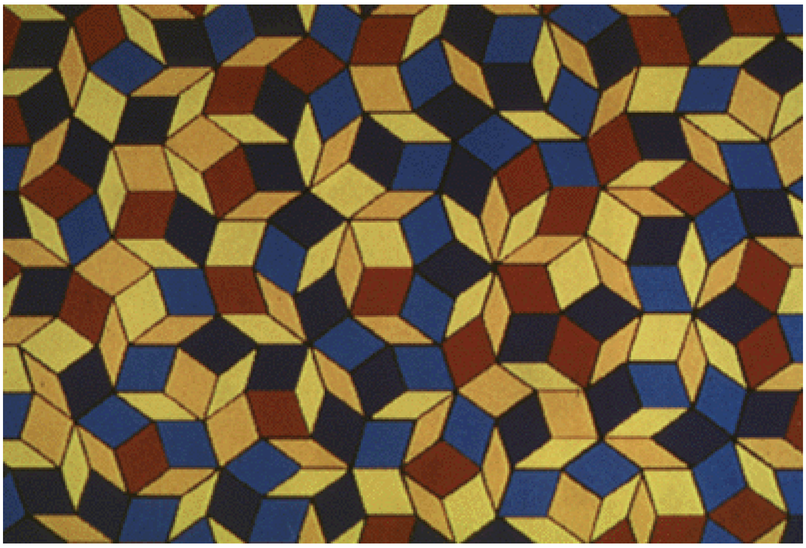
Another method of construction is given in Duret's *"Instruction in the Measurement with the Compass and Ruler of Lines, Surfaces and Solids," 1525.*

Its the same construction as given in *Geometria Deutsch,* a German book of applied geometry for stonemasons and

Golden Ratios in the Pentagram and Pentagon

The pentagon and pentagram are also interesting because they are loaded with Golden ratios, as shown in Boles p.48.

Golden Triangle



Slide 5-28: Emmer, plate F3
Emmer, Michele, Ed. *The Visual Mind: Art and Mathematics.* Cambridge: MIT Press, 1993.

The Golden Triangle

A golden triangle

also called the sublime triangle, is an isoceles triangle whose ratio of **leg to base** is the golden ratio.

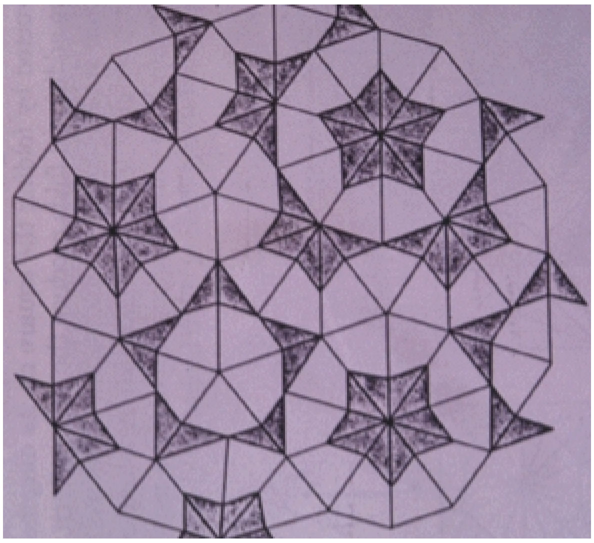
It is also an isoceles triangle whose ratio of **base to leg** is the golden ratio, so there are **two types**: Type I, acute, and type II, obtuse.

A pentagon can be subdivided into two obtuse and one acute golden triangle.

Euclid's Construction

Euclid shows how to construct a golden triangle. Book IV, Proposition 10 states, *"To construct an isoceles triangle having each of the angles at the base the double of the remaining one."*

Penrose Tilings



Slide: 5-27: Penrose Tilings.

Kappraff, Jay. *Connections: The Geometric Bridge between Art & Science.* NY: McGraw, 1990. p. 195

One place that the golden triangle appears is in the Penrose Tiling, invented by Roger Penrose, in the late seventies. The curious thing about these tilings is they use only two kinds of tiles, and will tile a plane *without repeating the pattern.*

Making a Penrose Tiling

A Penrose tiling is made of two kinds of tiles, called *kites* and *darts.* A kite is made from two acute golden triangles and a dart from two obtuse golden triangles, as shown above.

Slide 5-29: NCTM Cover

Conclusion

So we covered the triangle, pentagon, and hexagon, with sides 3, 5, and 6. We'll cover the square and octagon in a later unit.

Its clear that these figures, being visual, carried even more powerful emotional baggage than the numbers they represent.

Next time we'll again talk about polygons, in particular the triangle. But I won't waste your time with some insignificant and trivial fact about the triangle, but will show that, according to Plato, triangles form the basic building block of the entire universe!

—

Reading

Joseph Campbell, *The Power of Myth,* pp. 25-29
Carl Jung, *Man andHis Symbols,* pp. 266-285
Euclid, *Elements,* V2, pp. 97-104
Kappraff, *Connections,* pp. 85-87, 195-197
Fisher, p. 92-94

Projects

The PLATONIC SOLIDS

"Let no one destitute of geometry enter my doors."

Plato (c. 427 - 347 B.C.E.)

Slide 6-1: RAPHAEL: ***School of Athens***
American Catalgo, p. 126, #21061. Fresco, Vatican, Stanza della Signurata, the Pope's Private Library

We now move forward in time about 150 years, still staying in Greece, from Pythagoras to Plato, himself a Pythagorean.

In our last unit we studied some polygons, and I said that one of these, the triangle, was thought by Plato to be the building block of the universe. He presented that idea and others about creation, such as the universe being created to resemble a geometric progression, in one of his books, the *Timaeus*.

In the *Timaeus*, we'll see how Plato describes how triangles make up five solids, now called the *Platonic Solids*, and how these solids make up the four elements and heaven. We'll look at regular polyhedra in general, and see why only five are possible.

Finally we'll see how the Platonic solids were used as art motifs even before Plato, how they were used later, and how the served to tie together three Renaissance mathematicians and artists, Piero della Francesca, Luca Pacioli, and Leonardo da Vinci.

Plato

Slide 6-2: RAPHAEL: *School of Athens.* Center section

Profile: Plato (c.427-347 B.C.E.) was born to an aristocratic family in Athens. As a young man Plato had political ambitions, but he became disillusioned by the political leadership in Athens. He eventually became a disciple of Socrates, accepting his basic philosophy and dialectical style of debate, the pursuit of truth through questions, answers, and additional questions. Plato witnessed the death of Socrates at the hands of the Athenian democracy in 399 BC. In Raphael's *School of Athens* we see Socrates prone, with cup nearby.

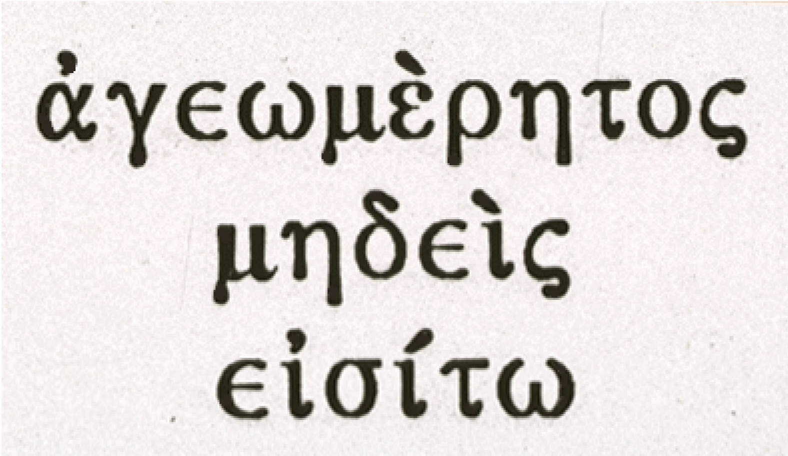
Plato's most prominent student was Aristotle, shown here with Plato in Raphael's *School of Athens,* Aristotle holiding his *Ethics* and Plato with his *Timaeus.*

Plato's Academy

In 387 BCE Plato founded an Academy in Athens, often described as the first university. It provided a comprehensive curriculum, including astronomy, biology, mathematics, political theory, and philosophy.

Plato's final years were spent lecturing at his Academy and writing. He died at about the age of 80 in Athens in 348 or 347.

Over the doors to his academy were the words



meaning, *"Let no one destitute of geometry enter my doors."*

Plato on Art and Geometry

Although Plato loved geometry, he would not have been good at teaching a course in Art & Geometry because he had a low opinion of art. He taught that, since the world is a copy or image of the real, then a work of art is a copy of a copy, at third remove from reality.

He writes in his Republic(p. 603),
"... painting [and] ... the whole art of imitation is busy about a work which is far removed from the truth; ... and is its mistress and friend for no wholesome or true purpose. ... [it] is the worthless mistress of a worthless friend, and the parent of a worthless progeny."

But on geometry he wrote in his Republic (p. 527),

*"[Geometry is] . . . persued for the sake of the knowledge of what eternally exists, and not of what comes for a moment into existence, and then perishes, ...
[it] must draw the soul towards truth and give the finishing touch to the philosophic spirit."*

The Timaeus

Plato left lots of writings. We've mentioned his *Republic* in our unit on number symbolism in which he gave the four cardinal virtues, but his love of geometry is especially evident in the *Timaeus.*

Written towards the end of Plato's life, c. 355 BCE, the *Timaeus* describes a conversation between Socrates, Plato's teacher, Critias, Plato's great grandfather, Hermocrates, a Sicilian statesman and soldier, and Timaeus, Pythagorean, philosopher, scientist, general, contemporary of Plato, and the inventor of the pulley. He was the first to distinguish between harmonic, arithmetic, and geometric progressions.

In this book, Timaeus does most the talking, with much homage to Pythagoras and echos of the *harmony of the spheres,* as he describes the geometric creation of the world.

Music of the Spheres

Plato, through Timaeus, says that the creator made the *world soul* out of various ingredients, and formed it into a long strip. The strip was then marked out into intervals.

First [the creator] took one portion from the whole	*(1 unit)*
and next a portion double the first	*(2 unit)*
a third portion half again as much as the second	*(3 unit)*
the fourth portion double the second	*(4 unit)*
the fifth three times the third	*(9 unit)*
the sixth eight times the first	*(8 unit)*
and the seventh 27 tmes the first	*(27 unit)*

They give the seven integers; 1, 2, 3, 4, 8, 9, 27. These contain the monad, source of all numbers, the first even and first odd, and their squares and cubes.

Plato's *Lambda*

Slide 8-72 : Arithmetic Personified as a Woman

Lawlor, Robert. Sacred Geometry. NY: Thames & Hudson, 1982. p. 7.

These seven numbers can be arranged as two progressions

Monad	1		Point
First even and odd	2	3	Line
Squares	4	9	Plane
Cubes	8	27	Solid

This is called *Plato's Lambda*, because it is shaped like the Greek letter lambda.

Divisions of the World Soul as Musical Intervals

Relating this to music, if we start at low C and lay off these intervals, we get 4 octaves plus a sixth. It doesn't yet look like a musical scale. But Plato goes on to *fill in* each interval with an arithmetic mean and a harmonic mean. Taking the first interval, from 1 to 2, for example,

Arithmetic mean = (1+2)/2 = **3/2**

The **Harmonic mean** of two numbers is the reciprocal of the arithmetic mean of their reciprocals.

For 1 and 2, the reciprocals are 1 and 1/2, whose arithmetic mean is 1+ 1/2 ÷ 2 or 3/4. Thus,

Harmonic mean = **4/3**

Thus we get the *fourth* or 4/3, and the *fifth* or 3/2, the same intervals found pleasing by the Pythagoreans. Further, they are made up of the first four numbers 1, 2, 3, 4 of the *tetractys*.

Filling in the Gaps

He took the interval between the fourth and the fifth as a full tone. It is

3/2 ÷ 4/3 = 3/2 x 3/4 = **9/8**

Plato then has his creator fill up the scale with intervals of 9/8, the tone. This leaves intervals of 256/243 as remainders, equal to the half tone.

Thus Plato has constructed the scale from *arithmetic calculations alone*, and not by experimenting with stretched strings to find out what sounded best, as did the Pythagoreans.

Project: Repeat Plato's calculations and see if you do indeed get a musical scale.

Forming the Celestial Circles

After marking the strip into these intervals, the creator then cut it lengthwise into two strips that are placed at an angle to each other and formed into circles. These correspond to the celestial equator and the ecliptic, the start of an *armillary sphere.*

Slide 10-121: Armillary Sphere

Turner, Gerard. Antique Scientific Instruments. Dorset: Blandford, 1980. p. 61

Recall our quotation from Plato's *Republic*, where, in the *Myth of Er* he wrote,

". . . *Upon each of its circles stood a siren who was carried round with its movements, uttering the concords of a single scale."* [Republic p. 354]

This is the origin of the expression, *Music of the Spheres.*

The Elements

The idea tthat all things are composed of four primal elements: **earth, air, fire, and water**, is attributed to **Empedocles** (circa 493-433 BCE), Greek philosopher, statesman, and poet. He was born in Agrigentum (now Agrigento), Sicily, and was a disciple of Pythagoras and Parmenides.

Remember the opposite forces, Yin and Yang, male and female, whose interaction created everything in the universe? Empedocles thought that active and opposing forces, love and hate, or affinity and antipathy, act upon these elements, combining and separating them into infinitely varied forms.

He believed also that no change involving the creation of new matter is possible; only changes in the combinations of the four existing elements may occur.

Empedocles died about 6 years before Plato was born.

The Universe as a Geometric Progression

Plato *deduces* the need for the four elements. *Timaeus*, 31B-32C

1. First, we must have fire, to make the world visible, and earth to make it resistant to touch. These are the two *extreme* elements, fire belonging to heaven and earth to earth. He writes,

. . . it is necessary that nature should be visible and tangible ...
and nothing can be visible without fire or tangible without earth ...

2. But two cannot hold together without a third as a bond. [like glue]

. . . But it is impossible for two things to cohere without the intervention of a third ...

3. And the most perfect bond is the connued geometric proportion.

... [and] the most beautiful analogy is when in three numbers,
***the middle is to the last as the first to the middle**, . . . they become the same as to relation to each other.*

4. But the primary bodies are solids, and must be represented by solid numbers (cubes).
To connect two plane numbers (squares) one mean is enough,
but to connect two solid numbers, **two means are needed.**

But if the universe were to have no depth, one medium would suffice to bind all the natures it contains. But . . . the world should be a solid, and solids are never harmonized by one, but always by two mediums.

Hence the Divinity placed water and air in the middle of fire and earth, fabricating them in the same ratio to each other; so that **fire might be to air as air is to water and that water is to earth.**

fire/air = air/water = water/earth

Thus the ratio is constant between successive elements, giving a geometric progression.

The Platonic Solids

The Platonic Solids belong to the group of geometric figures called *polyhedra.*

A **polyhedron** is a solid bounded by plane polygons. The polygons are called **faces**; they intersect in **edges**, the points where three or more edges intersect are called **vertices**.

A **regular** polyhedron is one whose faces are identical regular polygons. Only five **regular solids** are possible

cube tetrahedron octahedron icosahedron dodecahedron

These have come to be known as **the Platonic Solids**

The Elements Linked to the Platonic Solids

Plato associates four of the *Platonic Solid* with the four elements. He writes,

We must proceed to distribute the figures [the solids] we have just described between fire, earth, water, and air. . .

Let us assign the cube to earth, for it is the most immobile of the four bodies and most retentive of shape

the least mobile of the remaining figures (icosahedron) to water
the most mobile (tetrahedron) to fire
the intermediate (octahedron) to air

Note that earth is associated with the cube, with its six square faces. This lent support to the notion of the foursquaredness of the earth.

The Cosmos

But there are *five* regular polyhedra and only four elements. Plato writes,

"There still remained a fifth construction, which the god used for embroidering the constellations on the whole heaven."

Plato's statement is vague, and he gives no further explanation. Later Greek philosophers assign the dodecahedron to the ether or heaven or the cosmos.

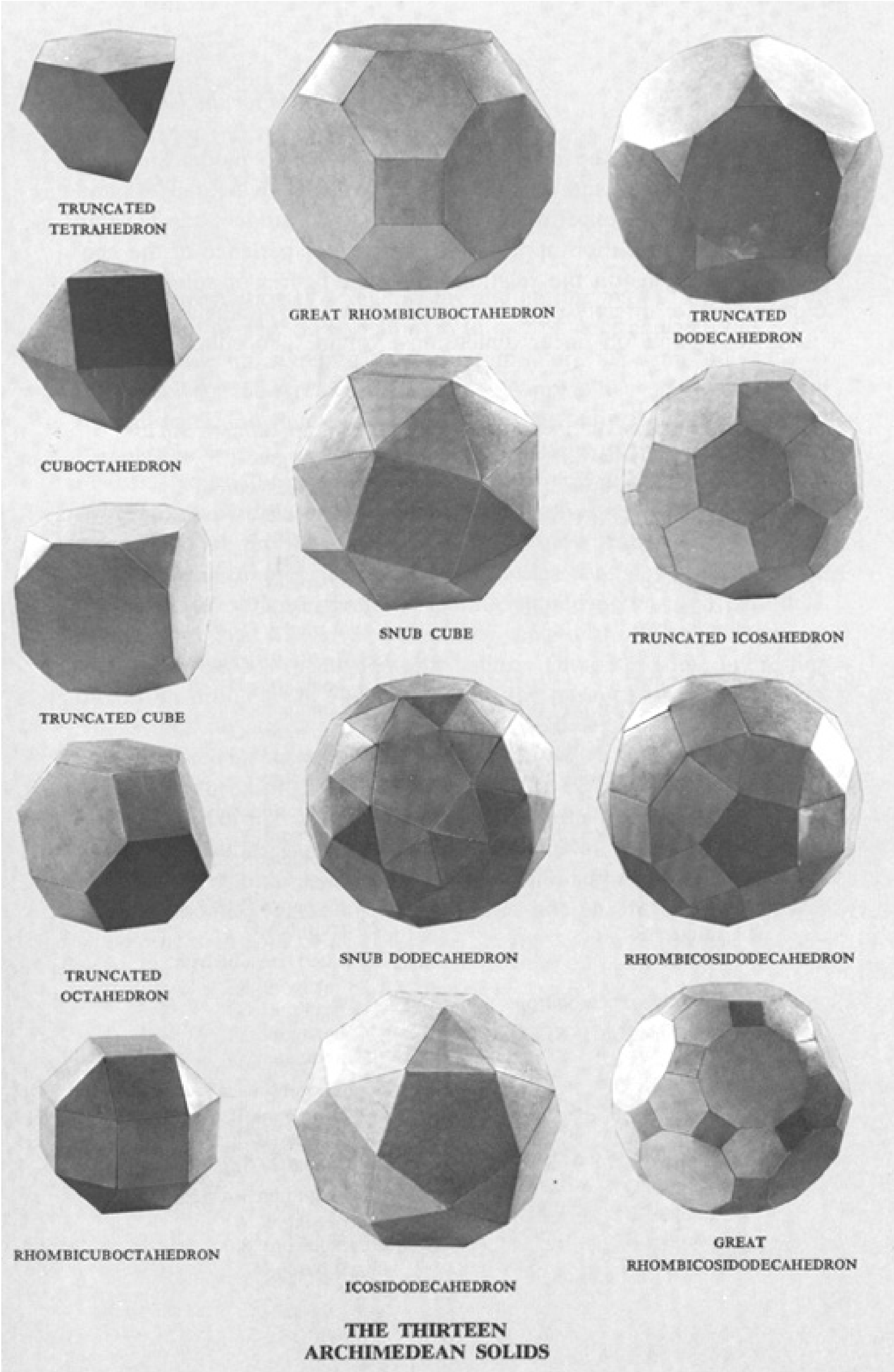
The dodecahedron has 12 faces, and our number symbolism associates 12 with the zodiac.

This might be Plato's meaning when he writes of ***"embroidering the constellations"*** on the dodecahedron.

Note that the 12 faces of the dodecahedron are pentagons. Recall that the pentagon contains the ***golden ratio.*** Perhaps this has something to do with equating this figure with the cosmos.

Other Polyhedra

The Archimedian Solids



Slide 6-4: Archimedian Solids

Wenniger, Magnus J. *Polyhedron Models for the Classroom.* NCTM 1966. p. 7

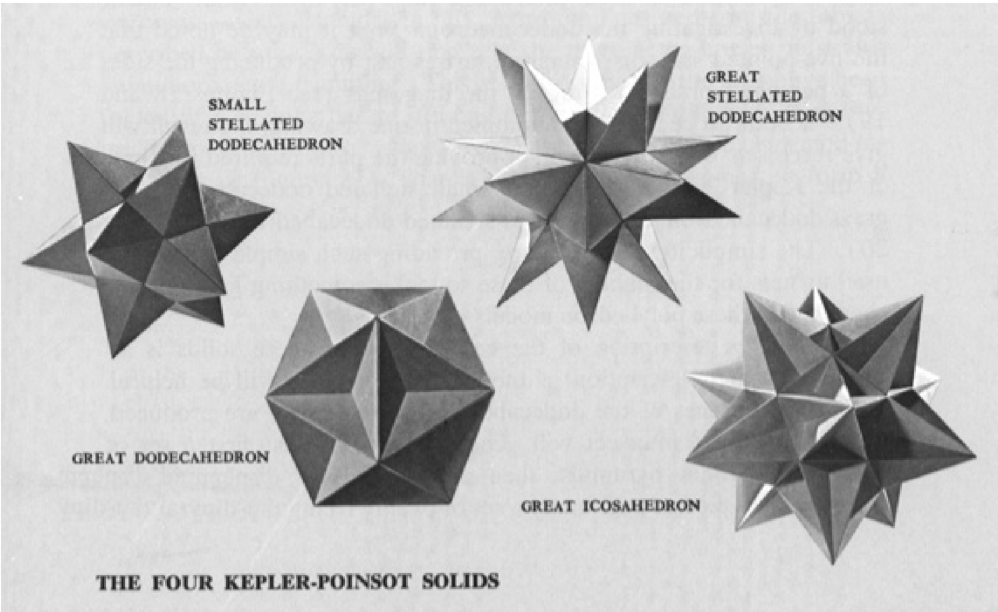
Other sets of solids can be obtained from the Platonic Solids. We can get a set by cutting off the corners of the Platonic solids and get *truncated* polyhedra.

They are no longer regular; they are called *semi* -regular; all faces are regular polygons, but there is more than one polygon in a particular solid, and all vertices are identical.

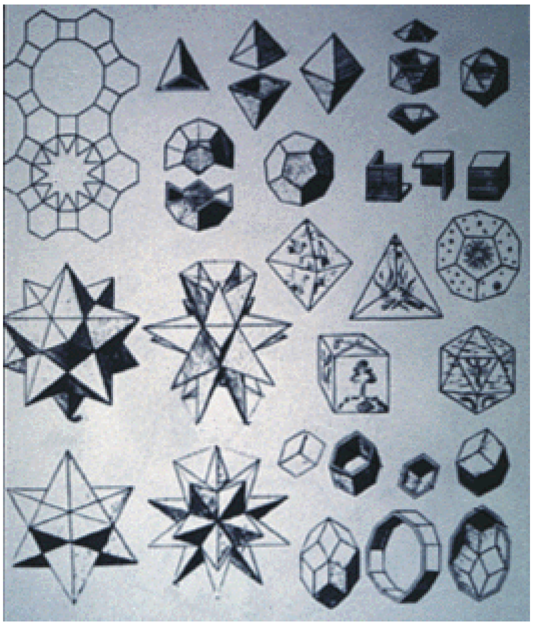
These are also called the *Archimedian Solids*, named for Archimedes, (287-212) the Greek mathematician who lived in Syracusa on the lower right corner of Sicily.

Mini-Project: Make some Archimedian Solids.

Star Polyhedra



Slide 6-5: The Four Kepler-Poinsot Solids
Wenniger, Magnus J. *Polyhedron Models for the Classroom.* NCTM 1966. p. 11



Slide 6-6: Engraving from *Harmonices Mundi*, 1619.
Emmer, Michele, Ed. *The Visual Mind: Art and Mathematics.* Cambridge: MIT Press, 1993. p. 218

The second obvious way to get another set of solids is to extend the faces of each to form a star, giving the so-called *Star Polyhedra.*

Two star polyhedra were discovered by Poinsot in 1809. The others were discovered about 200 years before that by Johannes Kepler (1571-1630), the German astronomer and natural philosopher noted for formulating the three laws of planetary motion, now known as Kepler's laws, including the law that celestial bodies have elliptical, not circular orbits.

Mini-Project: Make some star polyhedra.

Polyhedra in Art & Architecture

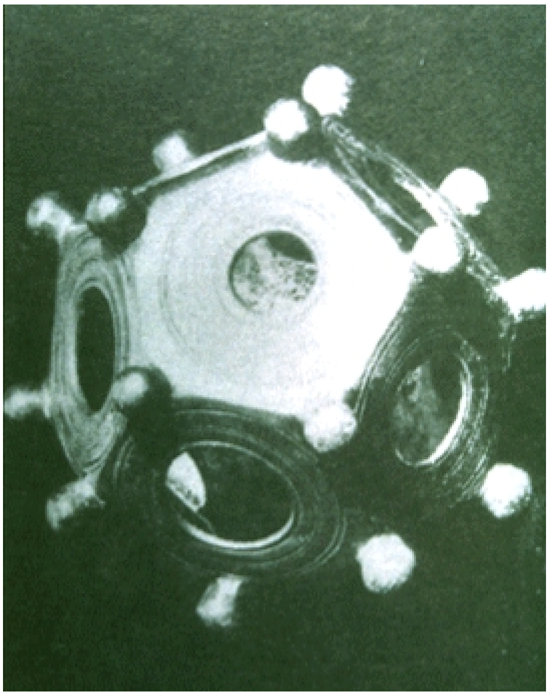
Polyhedra are Nothing New

Polyhedra have served as art motifs from prehistoric times right up to the present.

Slide 6-7: Pyramids

Tompkins, Peter. Secrets of the Great Pyramid. NY: Harper, 1971. Cover

The Egyptians, of course, knew of the tetrahedron, but also the octahedron, and cube. And there are icosahedral dice from the Ptolomaic dynasty in the British Museum, London.

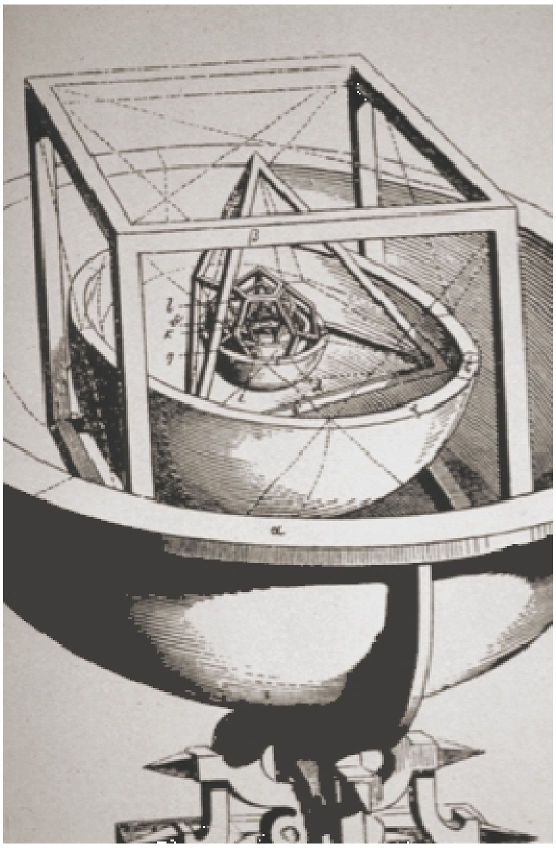


Slide 6-8: Etruscan Dodecahedron

Emmer, Michele, Ed. *The Visual Mind: Art and Mathematics.* Cambridge: MIT Press, 1993. p. 216

There were Neolithic solids found in Scotland, and excavations near Padova have unearthed an Etruscan dodecahedron, c. 500 BCE, probably used as a toy.

Kepler



Slide 3-6: Kepler's Model of Universe

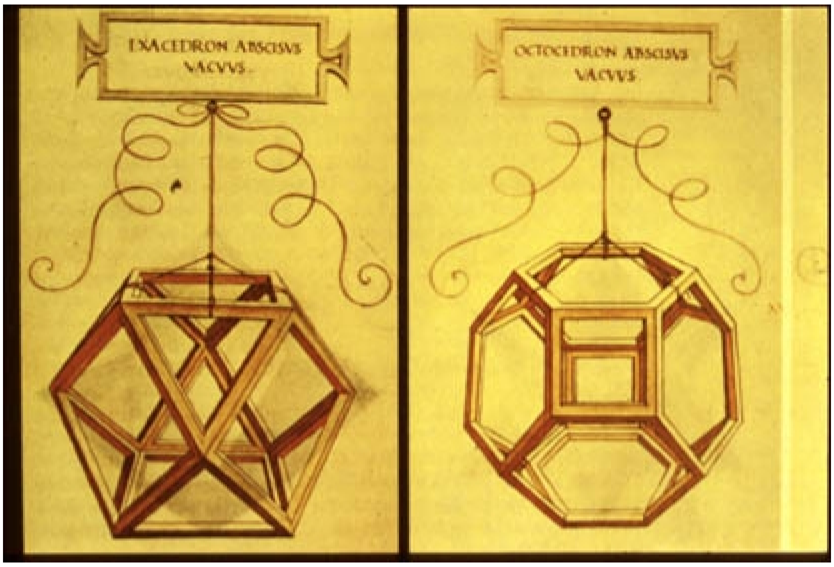
Lawlor, p. 106

In 1596 Kepler published a tract called *The Cosmic Mystery* in which he envisioned the universe as consisting of nested Platonic Solids whose inscribed spheres determine the orbits of the planets, all enclosed in a sphere representing the outer heaven. Of course, his observations did not fit this scheme. We'll encounter Kepler again in our unit on *Celestial Themes in Art.*

Polyhedra and Plagiarism in the Renaissance

Slide 14-10 : JACOPO DE 'BARBERI: ***Luca Pacioli***, c. 1499

This painting shows Fra Luca Pacioli and his student, Guidobaldo, Duke of Urbino. In the upper left is a rhombi-cuboctahedron, and on the table is a dodecahedron on top of a copy of Euclid's *Elements.*



Slide 15-11 : Leonardo's Illustrations for Luca's book.

Da Divina Proportione

Luca Pacioli wrote a book called *Da Divina Proportione* (1509) which contained a section on the Platonic Solids and other solids, which has 60 plates of solids by none other than his student Leonardo da Vinci. We'll tell the whole story of how this material was stolen from Luca's teacher Piero della Francesca in our unit on *Polyhedra and Plagiarism in the Renaissance.*

Platonic Solids as Art Motifs

Slide 6-12: UCELLO: *Mosaic from San Marco Cathedral, Venice,*1425-1430 plate J2

Emmer, Michele, Ed. *The Visual Mind: Art and Mathematics.* Cambridge: MIT Press, 1993.

Slide 16-08: DURER: *Melancolia I,* 1514

citation

Albrech Durer (1471-1528) had a keen interest in geometry, as we'll see in a later unit. This famous engraving shows an irregular polyhedron, as well as a sphere, a magic square, and compasses. People who have analyzed this polyhedron have decided that its actually a cube with opposite corners cut off.

Slide 6-13: NEUFCHATEL, Nicolaus: *Picture of Johannes Neudorfer and His Son,*1561.

Kemp, Martin. Leonardo on Painting. New Haven: Yale U. Press, 1989. p. 63

Slide 6-14: Gold-plated lion from the front of the *Gate of Heavenly Purity,* Closeup of Ball.

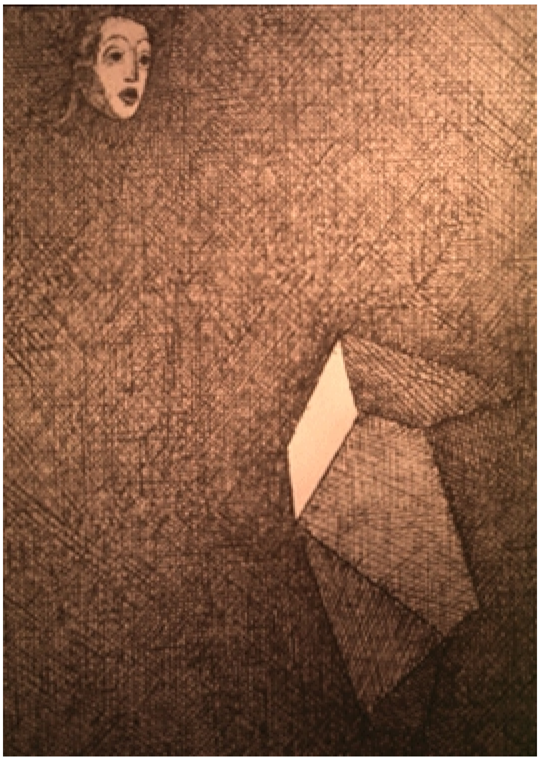
Forbidden City, Beijing. From Qing Dynasty (1736-1796)

This ball has hexagons interspersed with pentagons.

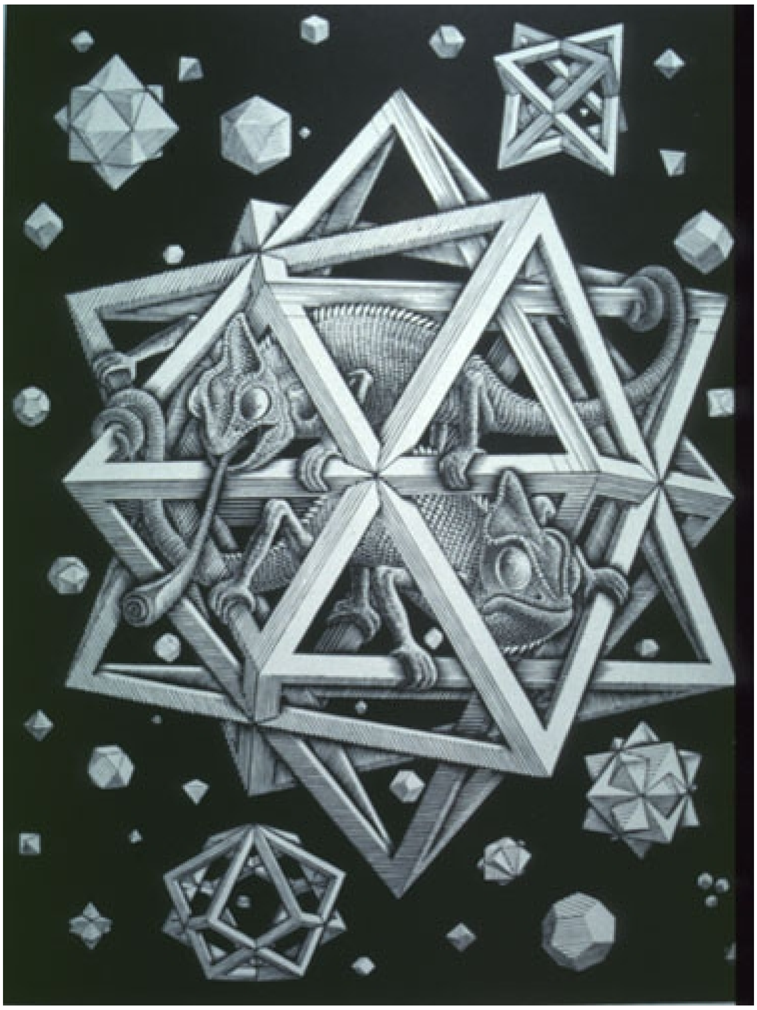
Polyhedra in Art in the Twentieth Century

Slides 6-15, 6-16, 6-17: Giacometti's Works

Hohl, Reinhold. Alberto Giacometti. NY: Abrams, 1972.



The Swiss artist Alberto Giacometti (1901-1966) often included polyhedra in his earlier surrealist works such as these two drawings and a sculpture.



Slide 21-5: ESCHER:*Stars*

1948 (#123)

We'll talk about M.C. Escher (1902-1972) in detail when we get to the 20th Century, but lets just peek at his 1948 engraving, *Stars.* Note the similarity between this polyhedron and Leonardo's illustrations for Pacioli's book.

Slide 21-06: Escher contemplating his nested set of Platonic Solids

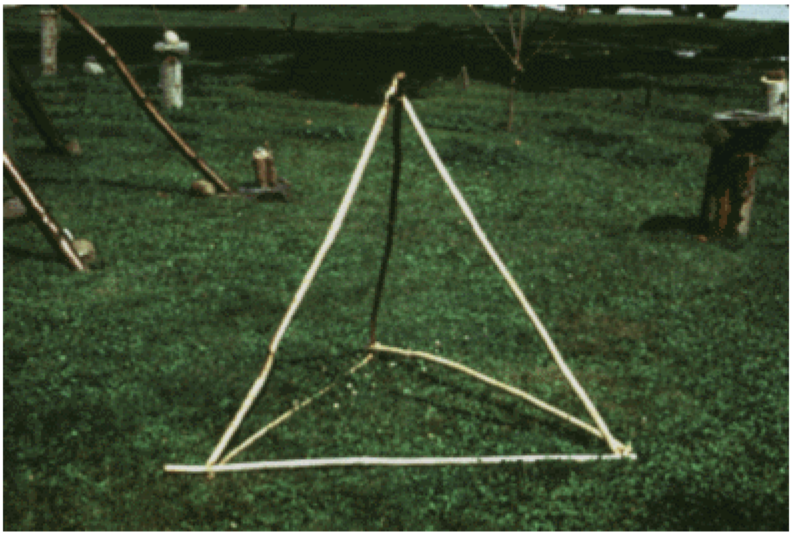
citation

Escher made a set of nested Platonic Solids. When he moved to a new studio he have away most of his belongings but took his beloved model.

Other Twentieth Century artists using polyhedra include Harriet Brisson, Paul Calter, and Lucio Saffaro.

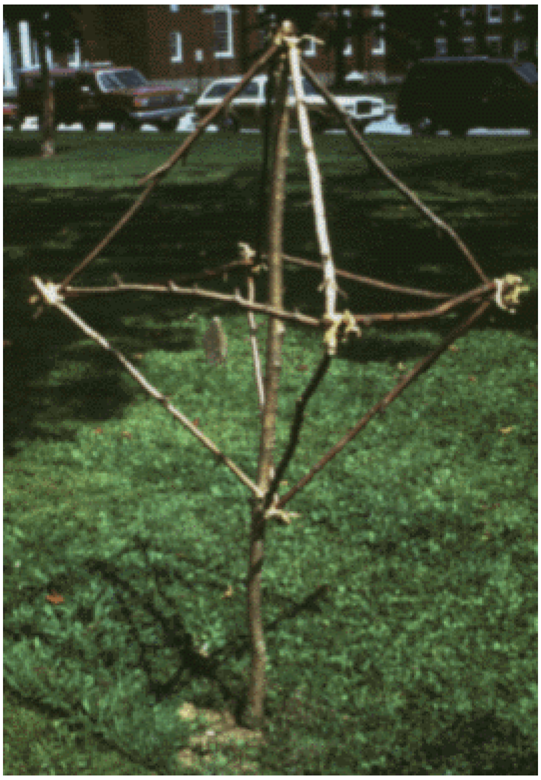
Slide 6-18:*Truncated Close-Packing Octahedra, Rhombidodecahedra, and Cubes. Plexiglass, aluminum tubes, and nylon cord, 1976*

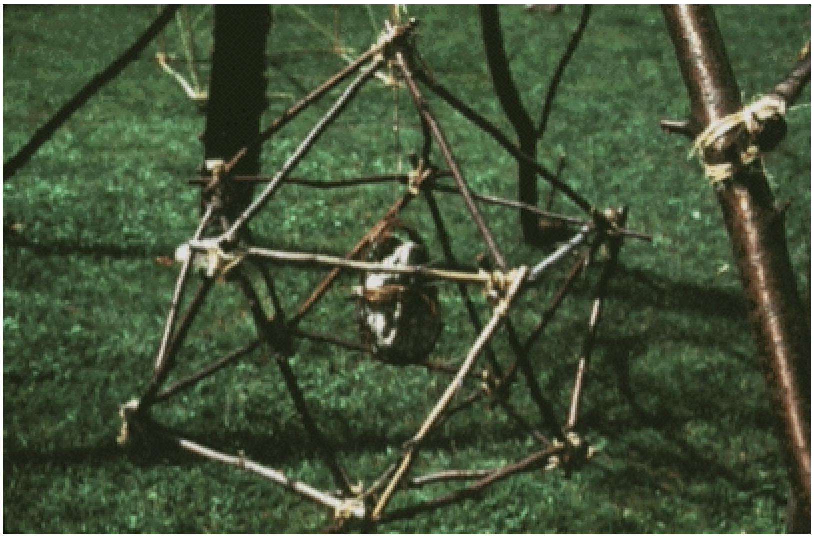
Emmer, Michele, Ed. *The Visual Mind: Art and Mathematics*. Cambridge: MIT Press, 1993. plate B3

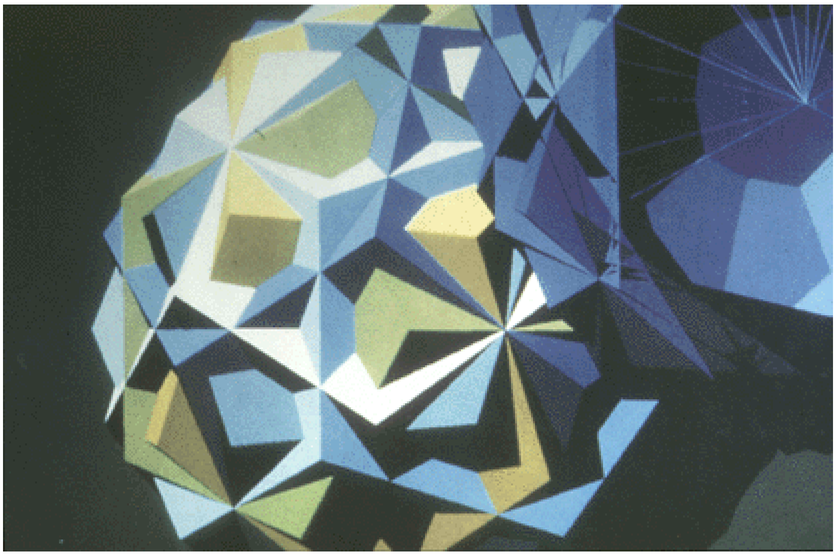


Slides 6-19, 6-20, 6-21: Platonic Solids

Calter's *Sorcerer's Circle*







Slide 6-22: LUCIO SAFFARO: *Platonic Forms.* Computer graphic, 1989.

Emmer, Michele, Ed. *The Visual Mind: Art and Mathematics*. Cambridge: MIT Press, 1993. plate A3

Project: Make a work of art featuring polyhedra.

Summary

So we've seen the origins of the Platonic Solids, starting even before Plato, and have briefly traced the influence of the polygons in art right up to the present.

We've also had a first look at some subjects we'll look at in more detail later.

For mathematical topics, we've briefly looked at sequences and series and the geometry of the polyhedra.

Reading

Reading Assignment:

Plato, *Timaeus*, the selection in your reader.

Emmer, *The Visual Mind*, from your reader

Calter, selection from *Technical Mathematics with Calculus*, handout

Additional References from your Bibliography :

Wenninger Pedoe Kappraff Irma Richter Lawlor Euclid

Ivins Newman Ghyka Wittkower Critchlow

Projects

Project: Repeat Plato's calculations and see if you do indeed get a musical scale.

Make some Archimedian Solids

Make some star polyhedra.

Make a work of art featuring polyhedra

Ad Quadratum, the Sacred Cut, & Roman Architecture

Slide 7-19: View of Pompeii with Mt. Vesuvius in the Background

Calter Photo

"Without symmetry and proportion there can be no principles in the design of any temple; that is, if there is no precise relation between its members, as in the case of those of a well shaped man."

Vitruvius, Book III, Chap. 1

Introduction

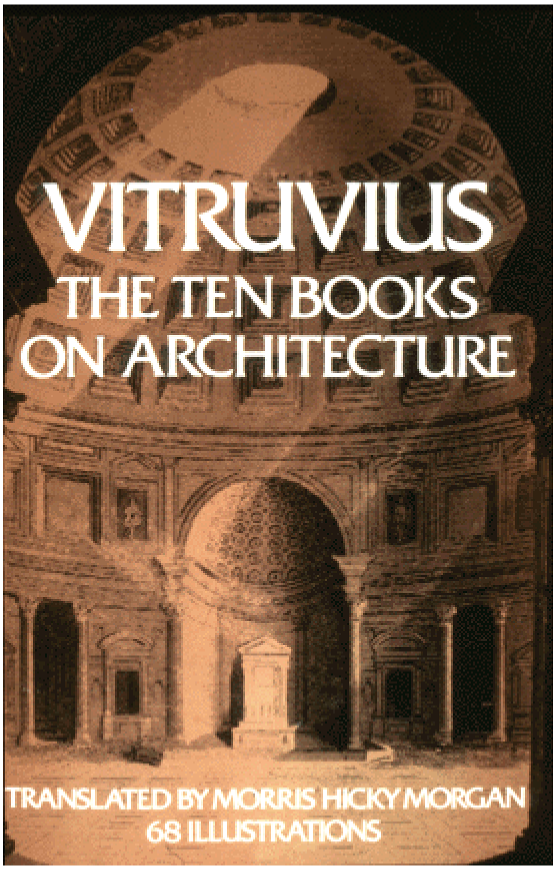
We leave ancient Greece now and move forward a few centuries to Rome. We'll start with a brief look at the architect Vitruvius, whose words echo down through the centuries.

When we discussed the polygons we looked at the triangle, pentagon, and hexagon, leaving the square and rectangle for this unit. We'll see here that the square and its diagonal, the so called *ad quadratum*, played a central role in Roman architecture.

Further, a geometric construction involving the square called *the sacred cut* also seems to have been used by the Romans. We'll see the sacred cut again when we discuss the octagon in the Middle Ages.

Finally we'll have a look at a particular rectangle, the so-called *roman rectangle*, thought to be central to the Roman's system of proportions.

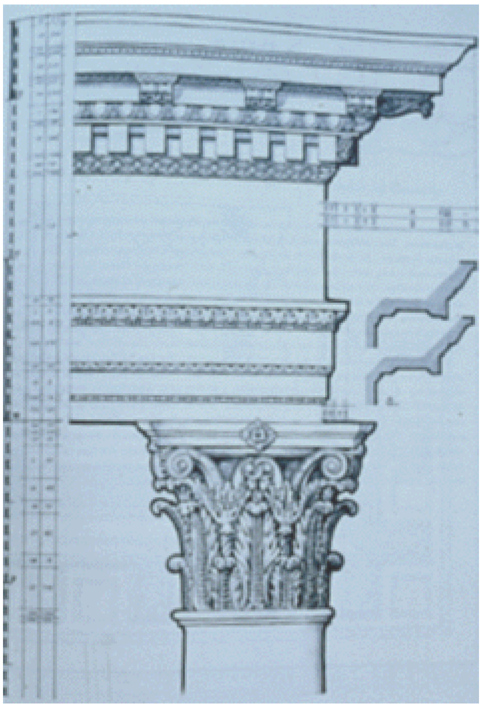
Vitruvius



Slide 7-1: Vitruvius Cover
Vitruvius. The Ten Books on Architecture. NY: Dover, 1960.

Vitruvius, whose full name is Marcus Vitruvius Pollio (70?-25 BC), was a Roman architect and engineer, born probably in Formiae (now Formie), Italy. He was an artillery engineer in the service of the first Roman emperor, Augustus.

The Ten Books on Architecture

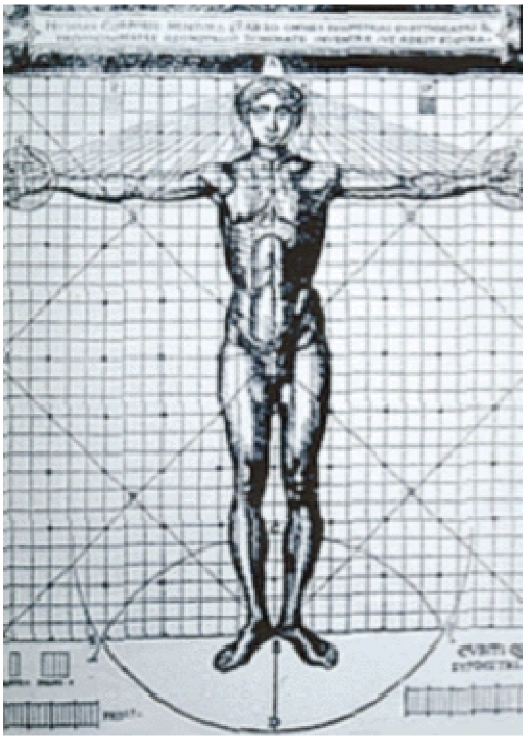


Slide 7-2: Corinthian Capital
Harris, Cyril. *Illustrated Dictionary of Historic Architecture.* NY: Dover, 1977.

His ten books on architecture, *De Architectura* (trans. 1914), are the oldest surviving work on the subject. They consist of dissertations on a wide variety of subjects relating to architecture, engineering, sanitation, practical hydraulics, acoustic vases, and the like. Much of the material appears to have been taken from earlier extinct treatises by Greek architects.

Vitruvius's writings have been studied ever since the Renaissance as a thesaurus of the art of classical Roman architecture. It's in Vitruvius that we first see the classical orders of architecture, Doric, Ionic, Corinthian.

Vitruvian Man



Slide 7-3: Vitruvian Man from Cesariano's edition of Vitruvius, 1521.
Wittkower, Rudolf. *Architectural Principles in the Age of Humanism.* NY:Random, 1965. Fig. 4.

In the Old Testament, we read in Isaiah,

"The carpenter stretches a line, he marks it out with a pencil; he fashions it with planes, and marks it with a compass; he shapes it into the figure of a man, with the beauty of a man, to dwell in a house."

Echoing this idea, In Book III of his *De architectura,* Vitruvius writes,

"Without symmetry and proportion there can be no principles in the design of any temple; that is, if there is no precise relation between its members, as in the case of those of a well shaped man."

He goes on to say,

". . . if a man be placed flat on his back, with his hands and feet extended, and a pair of compasses centered at his navel . . . the fingers and toes of his two hands and feet will touch the circumference of a circle described therefrom. And just as the human body yields a circular outline, so too a square figure may be found from it. For if we measure the distance from the soles of the feet to the top of the head, and then apply that measure to the outstretched arms, the breadth will be found to be the same as the height . . ."

This paragraph has given rise to many pictures of men in squares and circles, so called *Vitruvian men*, some as illustrations to editions of his book, including the famous one by Leonardo. We'll talk about Vitruvian men again in our units on Squaring the Circle and Renaissance architecture.

The Need for a System of Proportions

Architects and builders have always sought systems of proportions, and Vitruvius was no different. He wrote, *"Symmetry is a proper agreement between the members of the work itself, and relation between the different parts and the whole general scheme, in accordance with a certain part selected as standard."*

And later, *"Therefore since nature has proportioned the human body so that its members are duly proportioned to the frame as a whole, . . . in perfect buildings the different members must be in exact* ***symmetrical relations*** *to the whole general scheme".*

Here Vitruvius uses *symmetrical relationships* to mean *the same proportions*, rather than some kind of mirror symmetry. Such a system would use the repetition of a few key ratios, to insure harmony and unity.

It would have additive properties, so the whole could equal the sum of its parts, in different combinations. This would give a pleasing design, and maintain flexibility. Finally, since builders are most comfortable with integers, it would be based on whole numbers.

Three Systems of Proportions

In this course we'll cover the three main systems of proportion in architecture.

- 1. A system based on the musical ratios, used by Alberti and Palladio, which we'll cover in our unit on Renaissance architecture.
- 2. A system based on the golden ratio, such as Le Corbusier's Modulor, which we'll look at in our unit on twentieth century architecture.
- 3. A system based on the square, apparently used by the Romans, the subject of this unit.

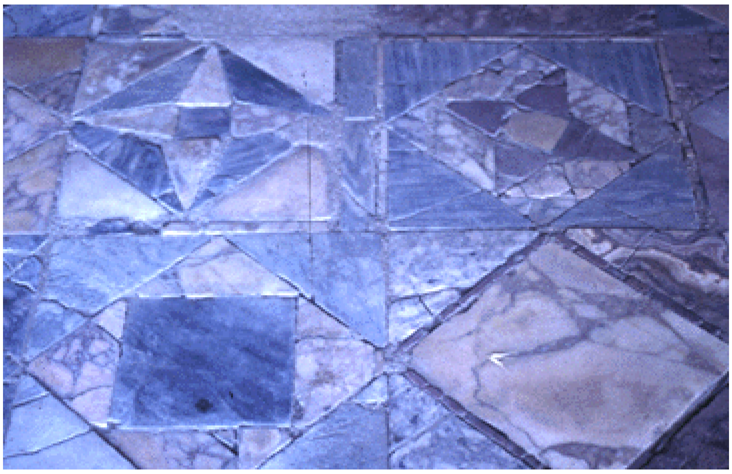
Ad Quadratum

Slide 7-11: Pisa Duomo

Calter Photo

Analysis of buildings at Pompeii and Herculaneum suggest that the design of the Roman house at all scales is based on the geometry of the square; they are said to be built *ad quadratum*.

The *ad quadratum* Figure



Slide 7-8: Pavement at Herculaneum

Calter Photo

In addition to underlying the ratios in some Roman buildings, this figure is often used for architectural decoration. I'll refer to this design, where one square is set diagonally inside another square, as the *ad quadratum* figure.

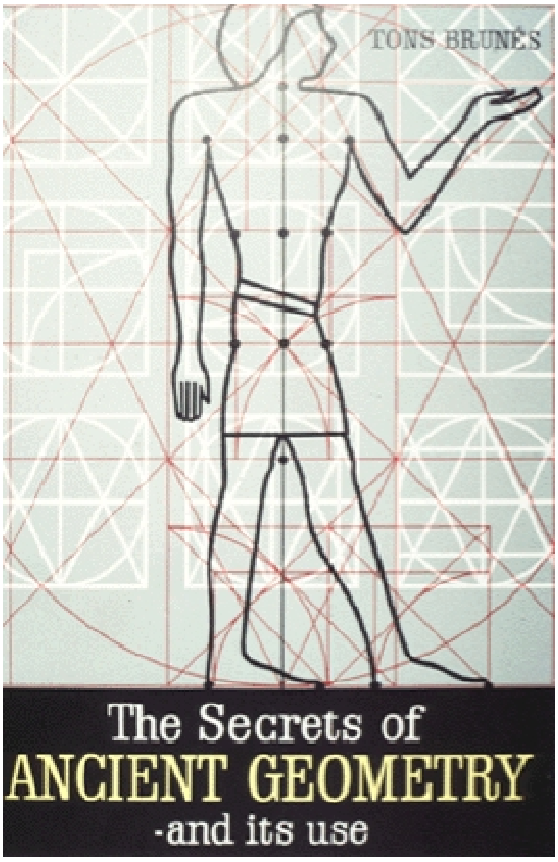
Root Two Rectangle

Vitruvius also mentions using the diagonal of a square as one way to find the dimensions of the atrium of a Roman house. *". . . by using the width [of the atrium] to describe a square* . . . *[and] drawing a diagonal in this square, and giving the atrium the length of this diagonal line."*

The Sacred Cut

In addition to the ad quadratum figure and the root-two rectangle, we have a third geometric system based on the square. It is called the *Sacred Cut*.

Tons Brunes



Slide 1-1: Secrets of Ancient Geometry and Its Use

Cover of Brunes' Book

This name was coined by the Danish Engineer Tons Brunes, in his book *The Secrets of Ancient Geometry and Its Use*. In that book he claims the sacred cut is found in the layout of many ancient building, including the Parthenon.

The Sacred Cut

Here is how the sacred cut is made.

- Draw a square.
- With a compass open to an amount equal to half a diagonal of the square, swing an arc with center at a corner of the square, passing through the center of the square and cutting two sides of the square.
- Repeat for the other three corners of the square.
- Through the eight points of intersection of the arcs and the sides of the square, draw four verticals and four horizontals.

The center square formed by this construction is called the *sacred cut square.*

Connecting the eight points of intersection of the arcs and the sides of the square consecutively forms an *octagon.*

Extending the Construction

The construction can be extended inward, by repeating the construction on the sacred cut square.

It can also be extended outward, joining the intersections of the circles and the diagonals, to form a square of which the original square is the sacred cut square.

Why Sacred?

Brunes calls this construction *sacred* because it contains both square and circle, uniting the earthly and the divine as in the Vitruvian man.

Furthermore, it *squares the circle*. The length of the four arcs equal the four diagonals of the half-square.

And, as mentioned, it gives the *octagon*, the shape universally used for baptistries and baptismal fonts.

Roman Architecture

We've done some interesting constructions. But what does this have to do with actual Roman Buildings? We'll show here that measurements of certain Roman buildings have show a preference for certain ratios, and that these ratios are found in the same constructions we have just done.

The Eruption of Vesuvius

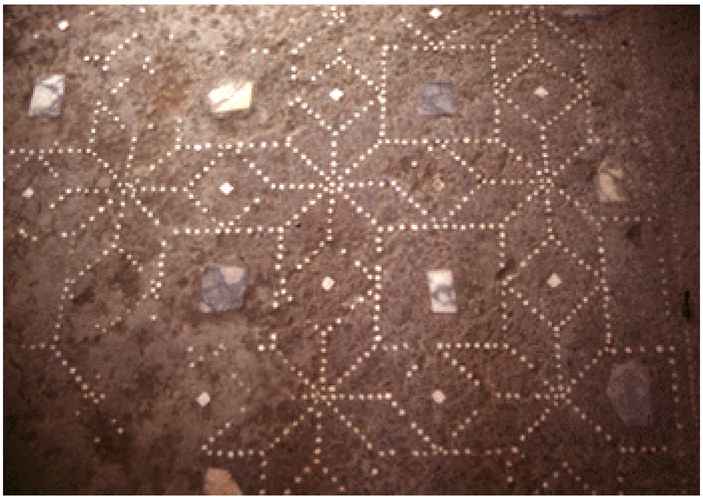
Slide 7-16: View of Pompeii with Vesuvius in Background

Calter Photo

Let's first go to southern Italy in the first century of the current era, to the coast south of Naples. Here **Mount Vesuvius** (Italian *Vesuvio,* from Oscan word *fesf,* "smoke") rises to a height of 4190 feet, the only active volcano on the European mainland.

On August 24 in the year AD 79 Vesuvius, the top of the mountain was blown off and the cities of Pompeii, Herculaneum, and Stabiae were buried, not by lava, but by ashes and mud. About 2000 people were killed.

Pompeii



Slide 7-20: Floor Pattern from Pompeii

Calter Photo

The mud went westward towards Herculaneum and the ash rained southeast and buried Pompeii, which lay undisturbed beneath the ashes for more than 1500 years. When excavations were started in 1748. it was seen how remarkably preserved everything was. The wet ashes and cinders had formed a hermetic seal about the town, preserving many public structures, temples, theaters, baths, shops, and private dwellings.

Herculaneum

Slide 7-23: Herculaneum, showing the depth of the ashes

Calter Photo

That same day Herculaneum was buried by mud about 65 ft deep. It is named for the person who, according to legend, founded the city; the mythical Greek Hercules. Like Pompeii it was a popular resort area for wealthy Romans.

Excavations of the ruins were begun at about the same time as at Pompeii. The diggers found many villas, a theater, marble and bronze sculptures, paintings, and a library of papyrus rolls in what was named the *Villa of the Papyri*, copied as the Getty Museum in Malibu.

House of the Tuscan Colonnade

Lets look at a single house in Herculaneum, *The House of the Tuscan Colonnade*. It was a typical house for the average patrician.

The ground plan drawn by Donald and Carol Watts shows the dimensions of the layout, in Oscan feet. (The Oscans were an early Italic people who lived in Campagna, the region around Naples. They built the original walls and towers of Pompeii and might have founded Herculaneum.) The dimensions found in the ground plan are:

5 12 17 29 41

Starting with 41 and calculate the sacred cut series by dividing by ($\sqrt{2}$ - 1) or (1 - $\sqrt{2}/2$) and rounding to integers, the Watts' get the same numbers found in the house of the Tuscan Colonnade.

The Garden Houses of Ostia

Slide 7-24: Aerial View of Ostia
Watts, Donald and Carol. *A Roman Apartment Complex.* Scien. Amer. Dec. 1986, p. 132-138.

Another site studied by the Watts' is at Ostia, near Rome. There they found the ground plan to totally conform to the sacred cut construction. We refer you to their Scientific American article, in the bibliography, for more detail.

Summary

Did the Romans really use this stuff? The measurements at Pompeii and Herculaneum are fairly convincing, and those at Ostia quite compelling, that these proportions were used at least in those cases.

We saw that the desirability of using a system of proportions, rather that picking each dimension with no regards to the others in a structure, was clearly stated by Vitruvius. But its hard to say if the use of these particular proportions based on the square was more widespread, because we have no written record verifying their use. Here, Vitruvius is vague.

There are pragmatic reasons to use a worked-out system of proportions. It saves time and energy to have standard proportions. It requires fewer decisions for home buyers. It gave them something that was safe, standard, and conforming to the general tastes. Perhaps builders had style books, the way we do now.

It was an easy way to insure proportional relations without having to do calculations. All the constructions can be done with straightedge and compass, both done with stretched cord, so it was easy for an on-site layout. Just drive pegs and swing arcs.

Reading

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NUMBER SYMBOLISM
in the
MIDDLE AGES

"But thou hast arranged all things by measure and number and weight."

Wisdom, XI, 20.

Slide 9-40: Chartres Cathedral

The great cathedrals are a symbol of the Middle Ages. By the Middle Ages numbers had acquired a metaphysical significance of their own, and were thought to be endowed with occult power. Thus they found their way into nearly every aspect of cathedral design, from the numbers of the pillars in the choir and layout of the facade, and, inevitably, to the division of the rose windows. In fact, the middle ages was nuts about numbers and geometry.

We've already looked at some roots in our unit on number symbolism: the Pythagoreans, astrology, the Old Testament, and mythology. Here we'll mention some new influences: the Cabbala and Gematria, the New Testament, neoPythagoreanism, neoPlatonism, Islam, alchemy, the Masons, Medieval magic, Tarot. As before, we'll do it *by the numbers*.

Introduction

Lets look at the period starting with the fall of the Roman Empire, around 476 CE, up to the start of the Renaissance in Italy, about 1400 CE, a period called the **Middle Ages**.

Painton Cowan writes that by the Middle Ages numbers had acquired a metaphysical significance of their own, and were thought to be endowed with occult power. It was a time of religious ferment, including early Christianity with its symbolism, Judaism with the Caballa and Gematria, Zoroastrianism, and Mithraism. These religions that required extensive symbolism, because things like the nature of God and the Trinity, hard to explain in words, could be expressed symbolically. A Medieval scholar like St. Augustine could say, *"No man can by force of will say that 3 times 3 is not nine."* So number and geometry found their way into nearly every aspect of cathedral design, from the numbers of the pillars in the choir and layout of the facade, to the division of rose windows.

In fact, the middle ages was nuts about numbers and geometry. But where did this emphasis on numbers and geometry come from?

Many symbols were mathematical in origin because math was considered an absolute.

We've already looked at some roots in our unit on number symbolism: Astrology, the Pythagoreans, Plato, Mythology, and the Old Testament, where in the Apocrypha is the well-known line,

. . . thou hast arranged all things by measure and number and weight." Wisdom, XI, 20.

Here we'll mention some new influences: the Cabbala and Gematria, neoPythagoreanism, neoPlatonism, Islam, Alchemy, and the Masons, Medieval Magic, Tarot, and the New Testament, where Matthew wrote, *"The very hairs of your head are all numbered"* (Matthew, 10:30).

One, the *Source*

Slide 8-3: Man in Sapphire Blue

Fox, Matthew. Illuminations of Hildegard of Bingen. Santa Fe: Bear, 1985. p. 22

Recall that for the Pythagoreans, *one* was the *monad*, the source of all other numbers, good, desireable, essential, indivisible. Pythagoras had much influence over Medieval philosophers such as Thomas Aquinas, who echoes the idea of *one* as a *source*. He wrote:

Since the soul is one and the powers are many; and since a number of things that proceed from one must proceed in a certain order; there must be some order among the powers of the soul.

But the number *one* is no longer an abstract first cause, as with the Pythagoreans, but is now the one God.

Two and Duality

We've seen earlier that *two* was the number of duality, opposites, and antithesis. To the Pythagoreans it was the *Dyad*, representing the Line, Diversity, a loss of unity, the number of excess and defect. It was their first feminine number.

Now in the Christian era, *two* became associated with

- the duality of the godhead: God the father, and Christ, and
- the two natures of Christ, human and divine.

Three and The Trinity

Slide 8-6: DURER: ***Trinity***
Lehner, Ernst. Symbols, Signs & Signets. NY: Dover, 1950 p. 109

But, according to Vincent Hopper, this duality was a problem. Remember that three, as we saw in our earlier, was the traditional number of deity, for example, for the Babylonians and Egyptians and for the triple goddess. Hopper cites the early theologians Clement, Origen, and Hipolytus, who lived in Alexandria and were steeped in number symbolism, who wrote that the duality of the Godhead: God the Father, and Christ was one weakness of Christianity.

This was overcome by the creation of the trinity by the addition of the Holy Ghost, giving the Holy Trinity, Father, Son, and Holy Spirit: Three persons in one god, probably the most well known of all triads and a major art motif. The trinity is mentioned in Matthew 28,19:

Go ye therefore, and teach all nations, baptizing them in the name of the Father, and of the Son, and of the Holy Ghost:

Three Epochs

Three is sometimes equated to the three days Christ spent in the tomb, and to the three epochs of one world,

1. Before the law (Adam to Moses)
2. Under the law (Moses to Jesus)
3. Under Grace (Christ to Last Judgement)

Three Theological Virtues

Slide 8-8: GUIDO RENI:*Charity* c. 1630.
Fisher, Sally. *The Square Halo.* NY: Abrams, 1995. p.102

Another trio from Christianity is Faith, Hope, and Charity. Its source is Paul's letter to the Corinthians, I Corinthians 13, 13.

And now abideth faith, hope, charity, these three; but the greatest of these [is] charity.

This trio can join our other trios of women; Three graces, three fates, three witches, three furies, three sirens, and also team up with Plato's four Justice, **cardinal virtues**, Prudence, Fortitude, Temperance, and to form the *seven virtues.*

The Triangle

Slide 8-10: ANTONIAZZO ROMANO, *God the Father*, detail. c. 1489.
Fisher, Sally. *The Square Halo.* NY: Abrams, 1995. p. 93

Most haloes are circular, but they come in other shapes as well. A triangular halo is used only for God the father, representing, of course, the trinity.

Four Evangelists

Slide 8-17: Four Evangelists from Book of Kells
Mackworth-Praed, Ben. The Book of Kells. London: Studio, 1993. Plate XIII

Groups of four in the New Testament include The Four Creatures of Revelation and The Four Horsemen of the Apocalypse. Also from the New Testament we have the four gospels written by the four evangelists; Matthew, Mark, Luke, and John. Three of them are often represented by animals; the ox for Luke, and the eagle for John, and the lion for Mark. Carl Jung says that the origin of the animal representations is Ezekiel 1, 5:

. . . of the midst there came the likeness of four living creatures. . . And . . . they had the likeness of a man. . . . And every one had four faces . . . the face of a man, and the face of a lion, on the right side: . . . the face of an ox on the left side; . . .they four also had the face of an eagle.

The Four Humours

Slide 8-25: Zodiacal Man

Wasserman, James. *Art and Symbols of the Occult.* Vermont: Destiny, 1993. p. 13

In the middle ages much was made of the four humours; phlegmatic, sanguine, choleric, and melancolic (phlegm, blood, bile, black bile). This picture combines the humours with the qualities "hot, cold, moist, dry" with a set of three zodiac signs in the four corners, which are also the cardinal points of the compass.

lower right	north	septentrionalia	Phlegmatic	sluggish, dull
lower left	south	meridionalia	Sanguine	hopeful, confident
upper left	east	orientalia	Choleric	hot-tempered, passionate
upper right	west	occidantalia	Melancolic	sad, depressed, melancholy

The four humours were also types of personalities or temperaments

phlegmatic, calm, stolid temperament; unemotional

sanguine, begnign and gentle, Cheerful; optimistic

choleric, easily angered; bad-tempered; irritable.

melancolic, named for a daughter of Saturn, is also called *saturnine.*

The Cross

The number four relates to the cross, with its four extremeties, and to the square with its four sides. Both suggest the four directions or cardinal points. In both their members are perpendicular.

The cross is said to symbolize Cross-roads, that place in which all things meet, and from which all things are possible.

Five Stigmata

Slide 8-45: A Crucifixion

Christiansen, Keith. Italian Paintings. NY: Levin Assoc. 1992. p. 115

The cross is associated with the number 4, but also with the number 5, if the intersection is included. It symbolizes the four directions and *here*. This 5-point interpretation of the cross connects with the other main symbolism of five in Christianity, the number of wounds or *stigmata* received by St. Francis on Mt. Averna. It is the subject of many paintings.

The Pentagram

Slide 8-46: Tombstone showing Order of Eastern Star Symbol.

Women's auxilliary of the Freemasons

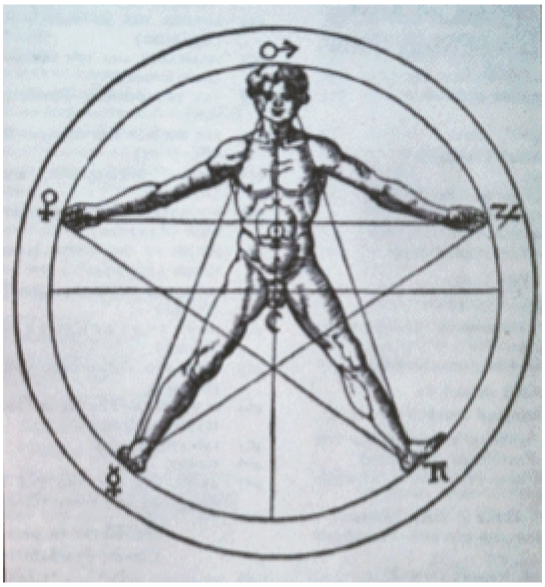
The geometric symbols of five are, of course, the pentagram and pentagon. As with the Pythagoreans, the pentagram was a symbol of recognition for certain secret societies, such as the Freemasons Flaming Star, and was supposedly on shield of Sir Gawain, one of the Arthurian Grail Knights.

Magic

Slide 8-47: Baphomet, illustraton for *Transcendental Magic* by Eliphas Levi, 1896.

Wasserman, James. *Art and Symbols of the Occult.* Vermont: Destiny, 1993. p. 70

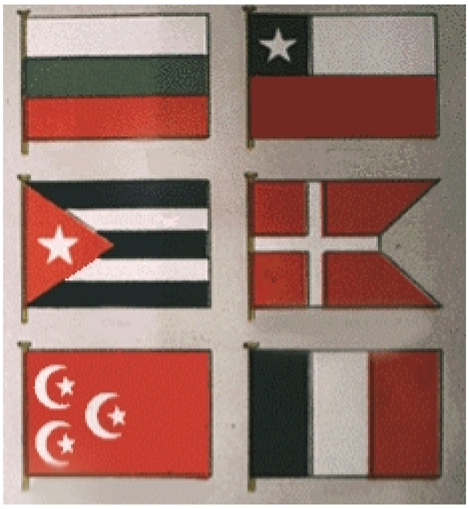
Unlike the cross, the pentagram was almost exclusively for magic, but that magic was strengthened by reference to the cross and the stigmata. In medieval magic the pentagram was called the pentacle, and became a symbol for man, the microcosmos.



Slide 8-48: Symbolic representation of man as Microcosmos. *Agrippa*.

Lehner, Ernst. Symbols, Signs & Signets. NY: Dover, 1950. p. 77

Flags



Slide 8-52: Flags showing the pentagram

The star pentagram appears on many flags. According to Betsy Ross' daughter, Washington and some others came to her mother's upholstery shop in Philly in June of 1776 with a rough draft of a flag. It had 6-pointed stars. Betsy showed them how to make a 5-pointed star by folding a piece of paper and making one cut with the shears. Washington then changed the design.

This story was later discredited. But it is interesting to note that George Washington studied engineering, geometry, trigonometry and surveying. His family crest contained stars and stripes, and some say that this may have influenced the flag design. Also, Washington was a Freemason.

....

Hands-On: Fold a pentagram by Betsy Ross' method.

Seven Virtues and Seven Vices

Slide 8-61: Gluttony

Art Bulletin. June 1996, p. 249

We've already mentioned Faith, Hope, Charity, the three Theological Virtues from I Corinthians in the New Testament, and Prudence, Fortitude, Temperance, Justice, the four Cardinal Virtues from Plato's Republic. These combine to make the *Seven Virtures,* one of the best-known heptads of Christianity.

To offset the virtues we have the Seven Vices, or Seven Deadly Sins, Gluttony, Lechery, Avarice, Luxury, Wrath, Envy, and Sloth. They are common art motifs.

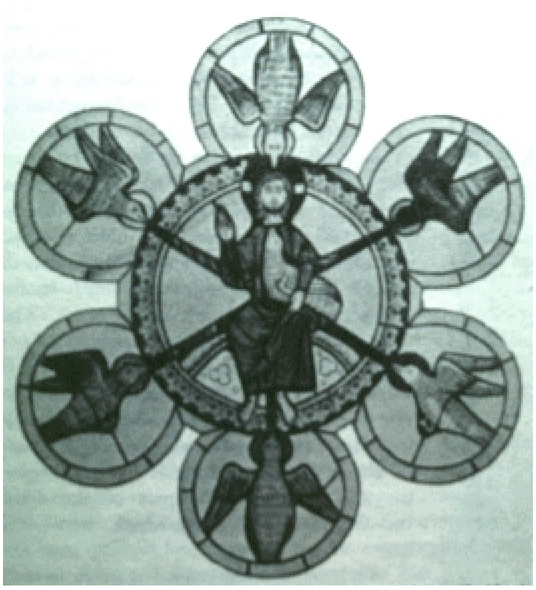
Slide 8-67: GIOTTO: *Scrovegni Chapel Frescos.*
citation

The seven virtues and seven vices are sometimes shown in opposition. In the Scrovegni chapel, the Last Judgement shows God with his right hand palm up towards the saved, and along the right wall are the seven virtues. His left hand is palm down towards the damned, and along the left wall are the seven vices, each opposite its corresponding virtue.

Prudence	vs	Foolishness
Fortitude	vs	Inconstancy
Temperence	vs	Ire
Justice	vs	Injustice
Faith	vs	Faithlessness
Hope	vs	Desperation
Charity	vs	Envy

The seven sins are sometimes shown as the seven-headed dragon of Revelation 12:3, a favorite motif of Durer's.

Seven Gifts of the Holy Spirit



Slide 8-58: LE MANS: *Christ with Seven Gifts of the Holy Spirit*

Male, Emile. *The Gothic Image.* NY: Harper, 1913. p. 167

Christianity has lots of groups of Seven. There are the Seven Sorrows of the Virgin and Seven Petitions of the Lord's Prayer. There are also seven gifts of the Holy Spirit, and Seven Joys of the Virgin, both popular art motifs.

Slide 8-59: VEIT STOSS: *The Annunciation.*1528,
St. Lorenz, Nurnberg.

Busch, Harald et al. *Renaissance Sculpture.* NY: Macmillan, 1964. p. 107

Seven Liberal Arts

Slide 8-71: Seven Liberal Arts on a cathedral

Throughout the Middle Ages, the *seven arts* represented the sum of human learning. The Medieval course of study consisted of:

The *trivium*, grammar, rhetoric, logic, for a Bachelor of Arts degree, and
The *quadrivium,* arithmetic, music, geometry, and astronomy, for a Master of Arts degree.

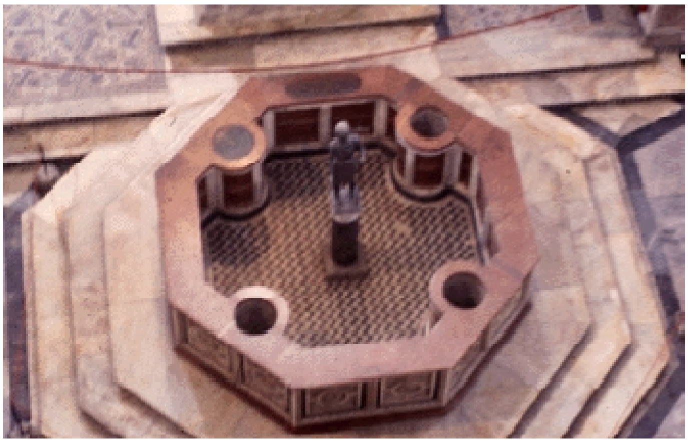
The quadrivium has its roots with the Pythagoreans, who gave us the very word *Mathematics,* and its branches.

The Seven Metals

Medieval alchemy identified seven metals. With their fondness for correspondences, these were assigned to the days of the week and the planets.

Day	Symbol	Planet	Metal
Sunday	☉	Sun	Gold
Monday	☽	Moon	Silver
Tuesday	♂	Mars	Iron
Wednesday		Mercury	Mercury
Thursday		Jupiter	Tin
Friday	♀	Venus	Copper
Saturday		Saturn	Lead

Eight and Baptism



Slide 8-80: Baptismal font in Pisa

Eight and the octagon represent resurrection and rebirth, because Christ rose from grave 8 days after entry into Jerusalem. Thus they became symbols of baptism, the spiritual rebirth of a person, and many baptistries and baptismal fonts are octagonal.

The Sacred Cut

An octagon can also be constructed in the following way:

Place a compass at each corner of a square and strike an arc which intersects the two adjacent sides and passes through the center of the square. This subdivides the square into a nine-module grid with modules of different proportions. Connecting the points where the arcs cut the square gives an octagon.

The Danish engineer Tons Brunes calles this construction the Sacred Cut. He claims it is the fundamental construction which forms the basis for system of geometry which has governed the construction of monuments in every period, from the Egyptian to the Medieval.

....

Hands-On: Construct an octogon by using the sacred cut.

Nine Ranks of Angels

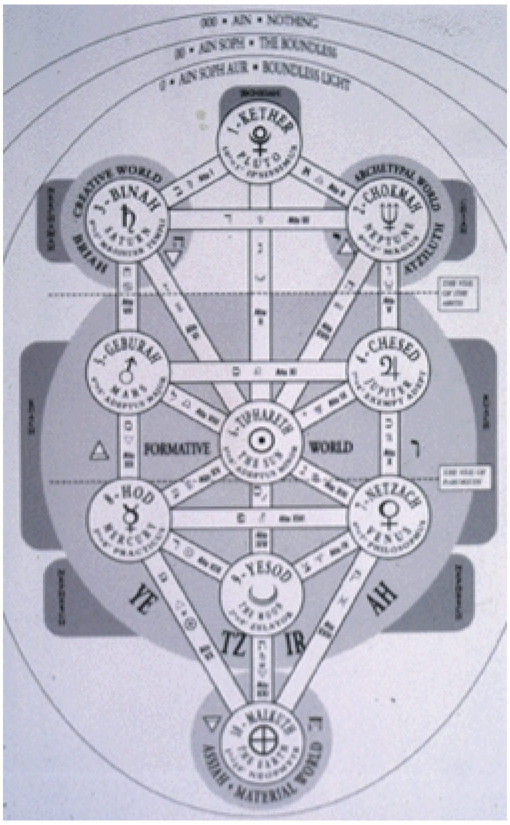
Slide 10-8: HILDEGARDE: ***Nine Ranks of Angels***

Hildegarde, p. 74

Nine is called the angelic number, after nine choirs of angels. We just spoke about the *Four Hierarchies of the Universe*. One of which was the *translunary world*, including the moon and everything above. The translunary world is divided into nine spheres, the empyrium, the stars, and the seven planets. Each had its own kind of angel; seraphims, cherubims, archangels, and the lot.

We'll introduce them all in our unit on Celestial Themes in Art.

Ten Spheres of the Sephiroth



Slide 8-104: Kabbalistic Tree of Life.
Wasserman, James. *Art and Symbols of the Occult.*
Vermont: Destiny, 1993. p. 33

In the Middle ages, we have the Ten Spheres of the Sephiroth. This figure also shows the Four Worlds that we discussed earlier, in light grey circles. It also shows the Sephiroth, which emanate or radiate from the divine power, each with its associated name, number and celestial body.

(1) Crown (2) Wisdom (3) Intelligence (4) Love

(5) Justice (6) Beauty (or Mercy) (7) Victory or Firmness (8) Splendor

(9) Foundation (10) Kingdom

Because the *sefiroth* were considered archetypes for everything in the world of creation, an understanding of their workings were felt to illuminate the inner workings of the cosmos and of history.

Twelve Apostles

From the New Testament, the main appearence of twelve is, of course, the 12 Apostles, or disciples. They are a popular art motif; on cathedrals, the 12 disciples are often arranged in 4 groups of three.

This also relates **twelve** to **seven**: both are composed of 3 and 4, one by multiplication and one by addition, and both 7 and 12 are important astrological numbers.

Thirteen and Bad Luck

Slide 15-27: Last Supper

The number of faithlessness and betrayal, the number of the twelve apostles plus Judas.

Forty and More Bad Luck

Temptation of Jesus

Slide 8-108: Duccio.

Fisher, Sally. The Square Halo. NY: Abrams, 1995. p. 69

Recall that forty was the number of trial and privation; days of the Great Flood, and the number of years Moses wandered in the desert. In the New Testament, after Jesus' baptism he was 40 days in the wilderness, tempted by the devil. The 40 days of lent commemorate this event.

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666 - The Number of the Beast

Probably the best known number from the Bible is the mysterious number of the Beast of Revelation.

Rev. 13, 18: *And here is wisdom. Let him that hath understanding count the number of the beast: for it is the number of a man; and his number is Six hundred threescore and six.*

Some interpreted the beast as the antichrist, Nero, Mohammed, or the Pope. The theologian Peter Bengus wrote a 700 page book, mostly devoted to 666, which he found equivalent to the name of Martin Luther. Luther replied with an analysis equating 666 with the duration of the Papal regime and rejoiced that it was so near its end.

Summary

So into Alexandria, the great mixing bowl, were poured the symbolism of the Greeks Pythagoras and Plato, along with Old Testament, and Oriental Mysteries like cults of Isis and Osiris, and Mithraism, with their astrology and number lore.

Out of Alexandria came sects like the gnostics, and forms of mysticism like the Cabbala with its number lore of Gematria, and the New Testament, itself mostly lacking in number symbolism, but where the gnostics found numerical secrets later amplified by Augustine, Hildegard, and Aquinas.

All these sources, along with the geometry from Islam, impregnated the Middle Ages with number symbolism, and number lore flourished wherever cosmic secrets were valued, as in astrology, Medicine, alchemy, magic, and the Tarot.

As with our earlier units on number symbolism and the polygons, we're on shaky ground with things that are ambiguous and poorly documented, and capable of multiple interpretations. But as before, we have tried to focus on those things that hve come down to us in literature, and especially in art motifs. The geometry and number lore that permeated the Middle Ages could not help but affect the most prominent architecture, the Gothic Cathedrals, and the Masons, to be covered in upcoming units.

Reading

Hopper, Butler, Carr-Gomm, Ferguson, Koch, Lehner, Hall, Sill.

Projects

Fold a pentagram by Betsy Ross' method

Fold an octagon, NCTM constructions 44 and 48

Construct an octogon by using the sacred cut.

Do a semiregular tiling using the octagon as one of the elements.

Find out what is meant by each number in *Green Grow the Rushes, Ho.*

The Circle, The Wheel of Fortune & The Rose Window

My heart leaps up when I behold ... A rainbow in the sky:

William Wordsworth (1770-1850)

Slide 9-1: Alfons Mucha (1860-1939). ***Slavia,*** **(1908) National Gallery, Prague**
VG Bild-Kunst, Bonn

Introduction

In an earlier unit we looked at the polygons, starting with the three-sided triangle and continuing to the eight-sided octagon. Now in our minds lets increase the number of sides to 9, 10, 20, 100, 1000, 1,000,000. Our polygon gets to look more and more like a circle. The circle is considered a symbol of unity, because all the regular polygons are embraced by the circle.

It is also the symbol of infinity, without beginning or end, perfect, the ultimate geometric symbol. Its a symbol of democracy and the preferred shape for an assembly of equals; the council circle, the campfire circle, and King Arthur's round table. The circle is also the easiest geometric figure to draw accurately, with stick and string or forked stick.

In this unit we'll examine the mathematics and the symbolism of the circle, and show how it was prominent in Gothic architecture, especially in the Rose window. We'll examine figures with circular boundaries like the vesica, and its use as art motifs. Finally, we'll combine circle with square for the highly symbolic squaring of the circle.

Symbolism of the Circle

The circle has many interesting associations and appears in art in a number of forms. Here, we'll look at the circle in the form of the halo, the rainbow, the ring, the wheel, and the circle dance.

The Circular Halo

Slide 9-2: *Madonna Enthroned.* Giotto c. 1310.
Janson, H.W. *History of Art.* Fifth Edition. NY: Abrams, 1995.

Recall that a halo is a zone of light behind the head of a holy figure, like the halos we sometimes see around the sun or moon. It may have the shape of a circle or triangle. Also recall that a triangular halo is used only for God the Father.

A circular halo is used for Christ, Mary, and saints. Mary's circular halo is usually elaborately decorated. The circular halo of a saint is usually plain.

Cruciform Halo

Slide 9-4: ***Florence Baptistry Ceiling***
Nova Lux Slide Set, Firenze

The Greek cross within a circle (cruciform nimbus) is used only when portraying Christ. In the San Giovanni ceiling, notice that the figure of God is also in the shape of a Greek cross, within a circle. Both can be taken as an examples of squaring the circle.

The Rainbow

Slide 9-6: ***Last Judgement, closeup.***Giotto, 1305. In Scrovegni chapel.

My heart leaps up when I behold, A rainbow in the sky:

William Wordsworth (1770-1850)

The circle has heavenly associations in the rainbow, which appears to touch both heaven and earth at the same time. The rainbow was often used as the Lord's throne and in scenes of the Last Judgement. When tri-colored, is associated with the Trinity.

This connection to the rainbow probably came straight from both Old and New Testaments:

Genesis Chap. 9, 13*"I set my bow in the cloud, and it shall be a sign of the covenant between me and the earth."*

Ezekiel 1, 26. *"Like the appearance of the bow that is in the cloud on the day of rain, so was the appearance of the brightness round him."*

Revelation 4, 3. From the vision of God on his throne, *". . .and round the throne was a rainbow"*

Revelation 10, 1. *"Then I saw another mighty angel coming down from heaven, wrapped in a cloud, with a rainbow over his head, . . ."*

Iris, Goddess of The Rainbow

Iris was the Greek goddess of the rainbow, a messenger of the gods, like Hermes. She descended to earth on a rainbow, which touched both realms, representing a communication between the heavenly and the earthly.

Iris appears often in Virgil's *Aeneid*, as in, *"The maiden Iris hurried on her way, along her rainbow with a thousand colors . . ."*

The Rainbow in Painting

Slide 9-8: ***The Rainbow.*** c. 1878. George Innes (1825-1894) (New Janson p. 715)
Janson, H.W. *History of Art.* Fifth Edition. NY: Abrams, 1995.

Innes is considered the leader of the American Barbizon School. He imbued his landscapes with a sense of divine presence and rainbows had a spiritual significance for him.

Other landscapes with rainbows include Millet's Spring, Constable's *Salisbury Cathedral*, and Turner's *Buttermere Lake.* Note how they give a spiritual feeling to the scene by connecting the earth with the heavens. This illustrates a recurring theme: ***The union of earthly and divine.***

The Ring as a Symbol of Union

Slide 9-13: ***The Marriage of St. Catherine***, Barna di Siena, c. 1360
Museum of Fine Arts, Boston, Slide Collection

As a symbol for eternity, the ring is used for betrothal and marriage. Betrothal rings were an old Roman custom. Wedding rings came into use later.

A bishop's ring signifies his union with Church. A Nun's ring signifies her marriage with Christ. This slide shows Catherine of Alexandria receiving a with a ring, symbolizing her marriage to God,

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The Ring as a Symbol of Authority or Status

This comes from the use of the *signet ring,* one used to make an impression indicating authenticity, which goes back to ancient Greece. Moreover, it is a symbol of *designated* authority, since a ring is easily passed to another.

In Rome, wearing of rings of various metals was strictly regulated. Citizen's rings were iron, and were forbidden to slaves.

Church rings show the ecclesiastical office of the wearer. Papal ring or *Fisherman's Ring*, bears image of St. Peter fishing. It is broken at Pope's death.

Magic Rings

Rings of Jasper or Bloodstone were worn by Egyptians for success in battle or other struggle.

The Koran says Solomon had a magic ring which could give him power over enemies, and transport him to a celestial sphere where he could rest from the cares of state.

Romans wore rings dedicated to the goddess Salus (Hygeia) engraved with a pentagram and a coiled snake, to ensure good fortune.

Rings made of nails from coffins or church doors were popular talismans in the Middle Ages for curing cramp and other disorders.

Other examples include *Reynard the Fox*, the wily hero of the medieval verse cycles known as the *Beast Epics*, claimed to have a magic ring bearing three mysterious Hebrew words, that would make him invisible and shield him from witchcraft. In *Orlando Furioso* by Ariosto, Ruggiero arrives on a Hippoghriff and places a magic ring on Angelica's finger to protect her. And of course, we have Tolkiens's *Lord of the Ring:*

Three rings for the Elven-kings under the sky,

Seven for the Dwarf-lords in their halls of stone,

Nine for Mortal Men doomed to die,

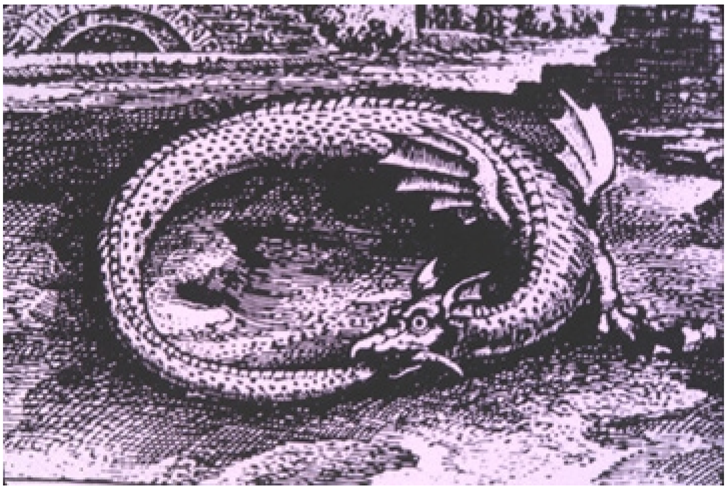
One for the Dark Lord on his dark throne

In the Land of Mordor where the Shadows lie.

One Ring to rule them all, One Ring to find them,
One Ring to bring them all, and in the darkness bind them

In the Land of Mordor where the Shadows lie.

Ouroboros



Slide 9-12: Serpent feeding on its own tail. Michael Maier, 1618. p. 101
Wasserman, James. *Art and Symbols of the Occult.* Vermont: Destiny, 1993.

The circle is the symbol for *infinity*, because the circle is endless, and may be considered a polygon with an infinite number of sides.

The snake or dragon with its tail in its mouth continually devouring itself and being reborn from itself is a symbol of eternity and of the cyclic nature of the universe. It expresses the unity of all things, which never disappear but change form in a cycle of destruction and re-creation.

It is also the alchemical symbol for chemical change. A dream about this serpent gave the chemist von Stradonitz the notion of the benzene ring, in the 19th century.

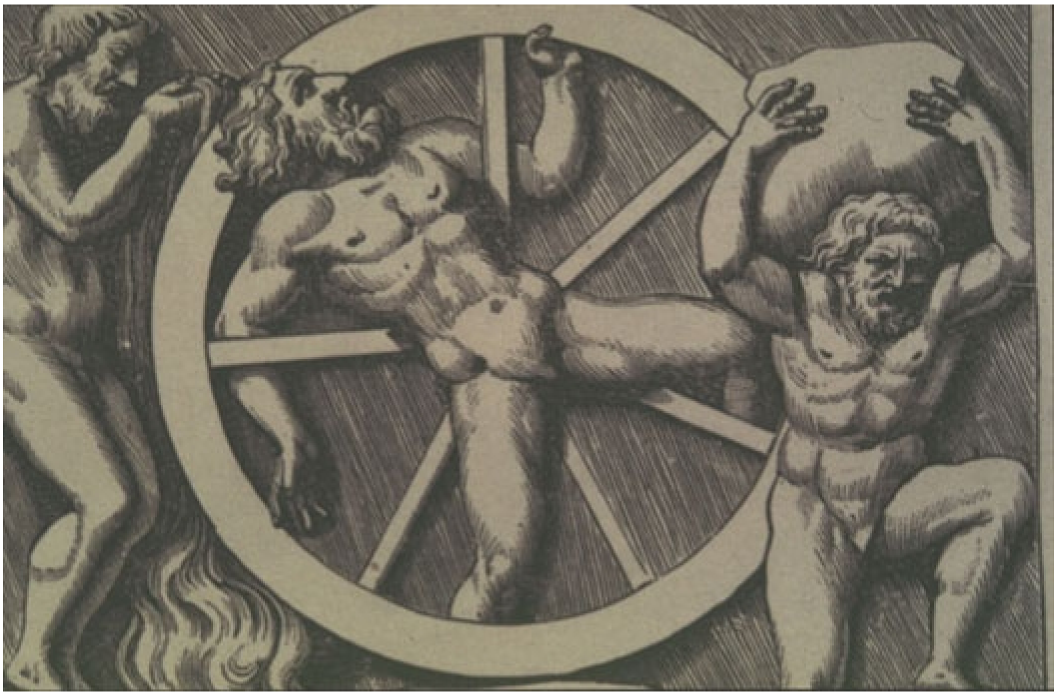
Catherine Wheel

Slide 9-17: Bartolomeo Barini (1450-91), Polyptych with St. James, Virgin, Child, & Saints, detail.

The circle is the symbol of **mobility**, in the form of a wheel. It also appears as an instrument of torment.

Catherine of Alexandria (d. 310 AD) refused to marry the emperor, and was imprisoned. Christ appeared and she wed him, a scene depicted in art as *The Mystic Marriage of St. Catherine.* Enraged, the emperor had a special spiked wheel designed for her torture, but the wheel broke and she was unharmed.

Ixion

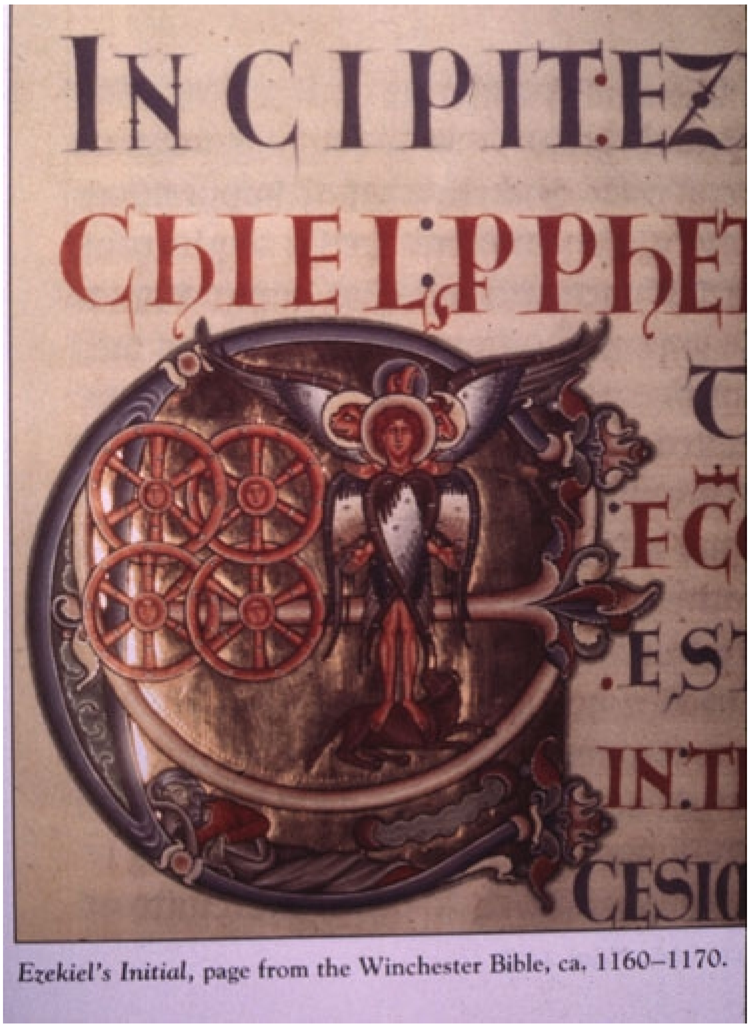


Slide 9-18: Cayley. *Classical Myths in English Literature.* Boston: Ginn 1893, p. 186

Ixion, in Greek mythology, was the first man to murder one of his kinspeople by killing his father-in-law to avoid giving him promised bridal gifts. He later tried to seduce Hera, the wife of Zeus.

As punishment, Ixion was bound to a wheel that revolved eternally in the underworld.

Ezekiel saw the Wheel



Ezekiel's Initial, page from the Winchester Bible, ca. 1160–1170.

Slide 9-19: ***Ezekiel's Initial.*** Page from the Winchester Bible, c. 1165. Campbell, Joseph, with Bill Moyers. *The Power of Myth.* NY: Doubleday 1988. p. 109.

There is a strange passage from The Book of Ezekiel that really excites all the flying-saucer enthusiasts.

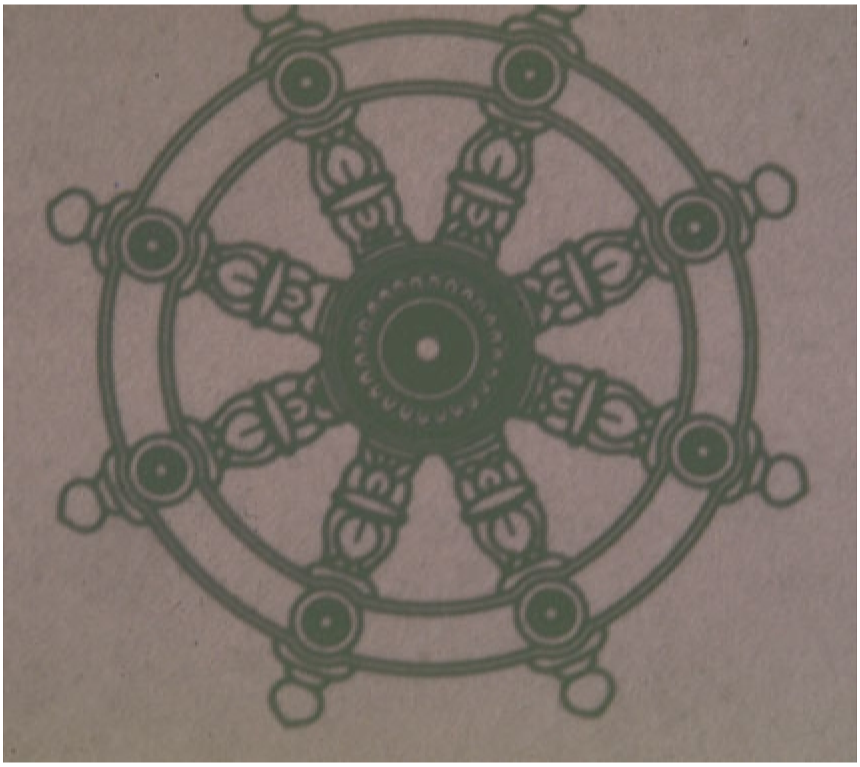
"*Now as I looked . . . I saw a wheel upon the earth beside the living creatures . . .*
their construction being as it were a wheel within a wheel . . .

The four wheels had rims and they had spokes,
and their rims were full of eyes round about.

And when the living creatures went, the wheels went beside them,
and when the living creatures rose from the earth, the wheels rose.

Wherever the spirit would go, they went, and the wheels rose along with them;
for the spirit of the living creatures was in the wheels."

The Wheel of Dharma



Slide 9-20: Wheel of Dharma

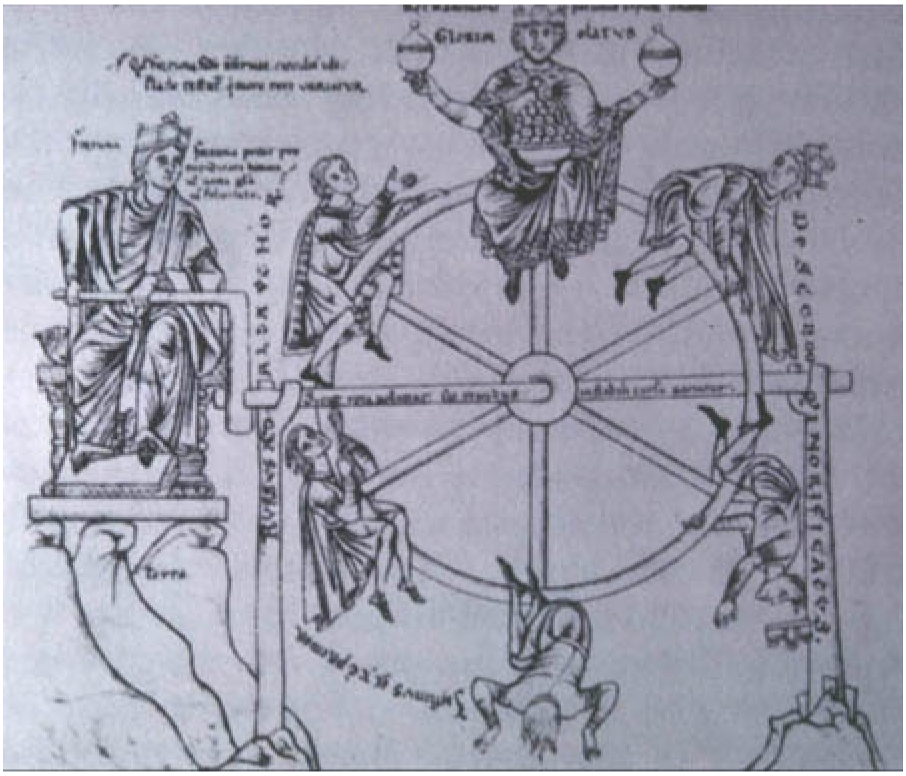
The *Wheel of Dharma* is a common symbol of Buddhism. Like the wheel of a cart that keeps turning, it symbolizes Buddha's teaching as it continues to spread endlessly. The eight spokes represent the eightfold Path of Buddha.

Prayer Wheel

Slide 9-21: Prayer Wheel. NG Feb. '80, p. 248
National Geographic Society

Here, a Tibetan woman spins an endless prayer written on a strip of paper coiled inside the cylinder.

Rota Fortuna, The Wheel of Fortune



Slide 9-23: Wheel of Fortuna in Hortus Deliciarum,
Kitzinger, Ernst. The Art of Byzantium and the Medieval West. Bloomington: Indiana, 1976. p. 351

The wheel, because it can *turn,* has often been associated with chance and fortune. In this picture, *Fortuna* is shown turning the crank on the wheel of fortune. Such depictions sometimes contained the legend.

Regnabo **Regno** **Regnavi** **Sum sine regno**

I shall reign *I reign* *I reigned* *I don't reign*

Slide 9-31: Dürer, *Fortune*, 1495, (Das Kleine Glück), Dürer, Albrecht. *The Complete Engravings, Etchings and Drypoints.* Ed. By Walter Strauss. NY: Dover, 1972. p. 14

The *sphere,* with its inherent instability, is also used to illustrate chance or fortune, as in this etching by Dürer.

Circle Dance

Slide 9-33: Shiva Nataraja, Lord of the Dance, South India, 11th Century
Campbell, Joseph, with Bill Moyers. *The Power of Myth.* NY: Doubleday 1988. p. 226

The circle dance, like the round table or council circle is democratic, where no one has a more prominent position than anyone else.

Slide 9-34: Poussin, *Ballo Della Vita Humana* Plate XL
Panofsky, Erwin. *Studies in Iconology.* NY: Harper, 1939.

Panofsky describes Poussin's, *Ballo della Vita Humana* as "*a kind of humanized Wheel of Fortune . . . where Poverty joins hands with Labour . . . Wealth . . . Luxury and dance to the lyre of Time . . .an infant plays with Time's hourglass and another blows soap-bubbles connoting transience and futility, while Sun drives his chariot through the zodiac.*"

The Rose Window

Numbers and the Medieval Cathedral

Slide 9-35: Milan Duomo Chartres Cathedral, slide # 511-A
Editions E. Houvret Slide Set

Medieval thinkers understood the mathematical aspects of number to be of divine origin. As part of the reason for this, Umberto Eco points to the triad of terms in the Book of Wisdom of Solomon, from the OT Apocrypha:

"*But thou hast arranged all things by measure and number and weight.*"
or
Numerus, pondus, mensura.

God The Geometer

Slide 9-37: ***God the Geometer,*** **Manuscript illustration.**
Clark, Kenneth. Civilization. NY: Harper, 1969. p. 52

The importance of number symbolism was matched by a dedication to geometry. Kenneth Clark points out that ". . . to medieval man geometry was a divine activity . . ."

According to Cowan, churches had been built on geometric principles since early Christian times. Geometry was the basis of all Gothic cathedrals, everything being created from basic relationships. We've seen that the ground plan was always cruciform, the baptism font always octagonal, and the baptistry itself often was, and the circle was everywhere.

This was symbolized in art by God holding a pair of compasses, a common motif in the Middle Ages. The art historian Ernst Gombrich credits a passage from the Old Testament as the inspiration for these portrayals. In Proverbs, Chapter 8 par. 27, *Wisdom* put forth *her* voice;

"When he established the heavens I was there: when he set a compass upon the face of the deep:"

The Gothic Style

Slide 9-39: Abbey Church of St. Denis Fig. 14
Simson. *The Gothic Cathedral.* NY: Harper, 1956

Gothic pertains to the Goths, who had nothing to do with this kind of architecture. It was a term of derision for any "barbarian" style not Greek or Roman. It started around 1140 around Paris, followed by the Age of the Great Cathedrals, 1150-1250. By 1250 had spread over most of Europe, by 1400 an *International Gothic* style prevailed.

We can precisely pinpoint the origin of the Gothic style in architecture; the rebuilding of the royal Abbey Church of St.-Denis, outside Paris. The structure appears light, graceful, weightless. Ribbed groined vaulting. Much larger windows. Pointed arches. The outward pressure of the roof is contained by heavy outside buttresses between the chapels.

Janson writes, *". . . the new spirit that strikes us at St. Denis is the emphasis on strict geometric planning and the quest for luminosity.* And Janson here paraphrases Abbot Sugar, who commissioned the work. *"Harmony (that is the perfect relationship among parts in terms of mathematical proportions or ratios) is the source of all beauty, since it exemplifies the laws according to which divine reason has created the universe.*"

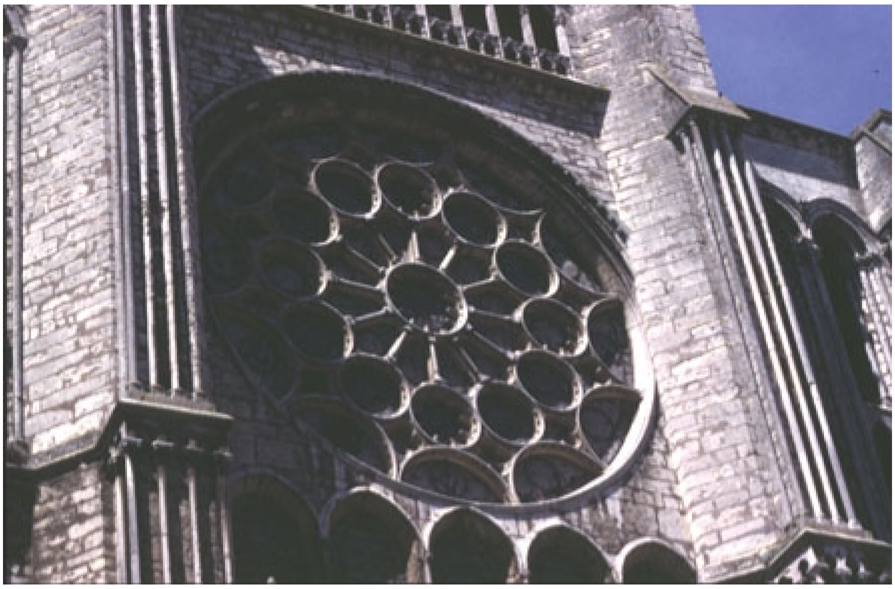
Chartres Cathedral of Notre Dame

Slide 9-40: Chartres Cathedral (1145-1220)
Campbell, Joseph, with Bill Moyers. *The Power of Myth.* NY: Doubleday 1988.

A year after St. Denis was finished work was started on rebuilding Chartres, and it was here that the Medieval craze for numbers and geometry seemed to reach its peak. According to Cowan, *"The scholars at Chartres were clearly fascinated by number and . . . geometry . . . as a key to understanding nature. Their preoccupation with numbers led to a trend of almost reducing theology to geometry."*

According to Eco, *"The School of Chartres remained faithful to the Platonic heritage of the Timaeus, and developed a kind of 'Timaeic' cosmology. For the School of Chartres, the work of God was order, opposite of the primeval chaos."*

The Rose Window



Slide 9-44: Rose Window Exterior

But the ultimate expression of the Medieval love of geometry and of the circle is the rose window, also called the Catherine window and the Wheel window. They look grand enough from outside the cathedral, but magnificent from inside, with sunlight streaming through.

Slide 9-46: Chartres North Window, Cowan Fig. 6

According to Painton Cowan, Rose windows use geometry in three different ways; *manifest, hidden, and symbolic.*

Manifest:*"That which makes the most immediate impact on the eye. . . the web of complexity and precision. . . each space defined by a yet smaller geometric figure - trefoil, quatrefoil, rosette, spherical triangle. . . within these can often be seen an even finer pattern woven into the glasswork . . . right down into every fibre and corner of the cosmic rose."*

Hidden: *"The secret geometry of the relationships and proportions of the parts."*

Symbolic: *"A kind of shorthand, where geometric figures represent different things."*

Influence of Astrology

Slide 9-48: The Towers

According to Painton Cowan, the astrological number twelve is the most common number in rose windows, especially in the south transepts. At Chartres there are three large rose windows, North, South, and West, each divided into twelve segments. The influence of astrology in Chartres also shows up in the zodiac signs over doorway on the west side, in a zodiac window, and in the towers of the sun and moon, in an outdoor sundial, and an astronomical clock.

Mandorla or Vesica

Another geometric figure made from circular arcs is the *vesica* or *mandorla*, a very common geometric figure in art history. A vesica is, simply, the common area to two overlapping circles.

We'll see that the vesica has several symbolic meanings and associations;

symbolic logic **almond** **aureole** **fish** **eye**

and we'll discuss them in that order.

Venn Diagram

In symbolic logic, this figure represents the intersection of two sets, a good symbol for the intersection of Art & Geometry.

The Vesica as an Almond

The vesica is also called the *mandorla* because of its almond shape, a powerful symbol because a nut is the seed from which a tree grows.

The almond is often mentioned in the bible . . .

Aaron's Rod in the Old Testament was made of almond wood. In Jeremiah, *". . . behold, the rod of Aaron for the house of Levi brought forth buds, and bloomed blossoms, and yielded almonds."*

In Exodus, the bowls of the menorah were to be made in the shape of almonds.

The Vesica as a Fish

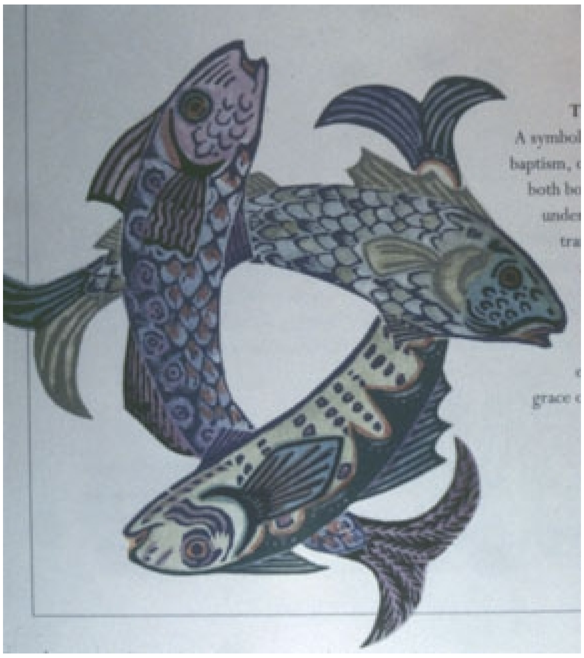
Slide 9-94: Car Medallion

Vesica pisces literally means *fish bladder* and if we extend the ends on one side, we get a simplified picture of a fish. The vesica is a variation of the zodiac sign for pisces, and thus has an astrological connection.

In Judaism, on the first day of Rosh Hashana Jews are encouraged to visit a body of water containing live fish and recite prayers in which we cast away our sins. "As fish depend on water so do we depend on God's providence." Also a fish's eyes never close, symbolizing God's unceasing watchfulness over us.

The fish is also a symbol for baptism. "Just as a fish cannot live without water, a Christian cannot live without the waters of baptism." A fish sometimes appears on the table in pictures of *The Last Supper.*

Fish in Greek is ΙΧΘΥΣ. These letters are also the initial letters of the words, *Jesus Christ God's Son Savior*. This association is often given as a reason why the fish is associated with Christianity.



Slide 9-97: Three Fish,
Prospero. *The Book of Symbols: Magic.* San Francisco: Chronicle, 1944. p. 22

A figure showing Three Entwined Fish has double symbolism, both of baptism and of the Trinity.

The Vesica as Aureole

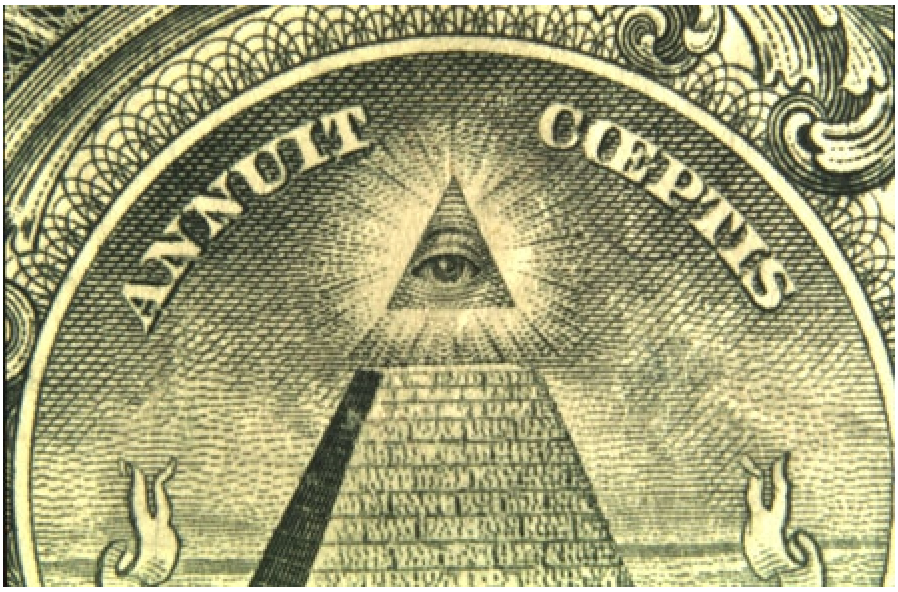
Slide 9-98: Christ Pantocrater Master of Tahull, c. 1123.
Fisher, Sally. *The Square Halo.* NY: Abrams, 1995. p. 91

The Middle Ages saw numerous appearances of the vesica used as an aureole, a field of radiance surrounding the entire body, a sort of body halo surrounding holy figures. This use continued into the Renaissance. It is the most common use of the vesica as an art motif. Interpreting the aureole as a Venn diagram, it could represent the uniting of God and man or the intersection of two realms, earthly and divine.

The Vesica as an Eye

Slide 9-105: Transfiguration. Mosaic. Monastery of St. Catherine, Mt. Sinai, Egypt. C. AD 560.
Hartt. *A History of Painting, Sculpture, Architecture* p 322.

As an aureole for a standing person, the vertical vesica seems a logical choice. But sometimes the vesica is *horizontal*. In such cases we get the impression of an eye, usually representing the omnipresent, all-seeing divinity.



Slide 9-108: Eye on Pyramid on Dollar Bill

Joseph Campbell writes, *"When you're down on the lower levels of the pyramid you will be on either one side or the other. But when you get to the top, the points all come together, and there the eye of God opens."*

Slide 9-109: Islamic Amulet,
Prospero. *The Book of Symbols: Magic.* San Francisco: Chronicle, 1944. p. 13

This Islamic amulet represents the eye as a spiritual gateway leading to the soul (the circle).

Slide 9-111: Etched Hourglass Nebula. NG Cover, April '97
National Geographic Society

In a recent National Geographic we read, *"Astronomers looked 8,000 light-years into the cosmos with the Hubble Space Telescope, and it seemed that the eye of God was staring back."*

Squaring the Circle

We just saw that the circle is the ultimate geometric figure, perfect, infinite, representing the divine, and we had seen that the square often represented mankind. Combining the two figures had special significance, *the reconciliation of the heavenly and infinite with the earthly and man-made.*

Recall from our unit on Egypt we said that the problem of squaring the circle is one of constructing, using only compass and straightedge;

(a) a square whose perimeter is exactly equal to the perimeter of a given circle, or

(b) a square whose area is exactly equal to the area of a given circle.

In that same unit we also saw that a circle whose radius is the pyramid height

1. has the same perimeter of the base of the Great Pyramid
2. and the same area as the rectangle whose width equals the pyramid's base and whose length is twice the pyramid height

Also recall the Sacred Cut construction for drawing the octagon. We said that, according to Brunés, the "sacredness" of the Sacred Cut lies in its very nearly solving the riddle of how to square the circle. The length of each arc equals the length of the diagonal of half the reference square, to within 0.6%. Thus a square of four such diagonals equals (approximately) the perimeter of a circle composed of four sacred cut arcs.

The Vesica and Squaring of the Circle

There's one more circle-squaring construction to show, that Critchlow claims was used in the layout of Hindu temples.

Draw two orthogonal vesicas. Then connect the four intersection points of the vesica to get a square. This square is approximately equal in perimeter to the original circle, a *squaring that circle.*

Neolithic Stone Circles

Slide 9-114: Castlerigg

We saw that Stonehenge is circular, but look at another stone "circle," Castle Rigg. The most striking thing about this ring is that it is *flattened*. In fact, many of the rings in Britain are flattened.

Why Flattened? Why go to the trouble of producing a flattened circle when a circular shape is so much easier to produce? Recall that the diameter of a circle will fit around the perimeter of the circle three times, plus a bit more, actually π times, where $\pi \approx 3.1416...$, an irrational number.

The speculation about why a stone circle was flattened was to make its perimeter an integral multiple of the radius drawn to their circular part of its perimeter.

Why? According to Critchlow, *"The constructions . . . were typical of those which numerically rationalize the perimeter of the primary circle. This balance between 'irrational' geometry and rational numbering is a fundamental reconciliation lying at the roots of sacred geometry. . . "The squaring of the circle is a . . . symbol [of] . . . the establishing of Heaven on Earth . . .."*

It was an attempt to *rationalize the irrational.*

The Molten Sea

Another example of a circle with a perimeter that is an integral multiple of the diameter is found in the bible. I KINGS, Chapter 7 reads like an installment of *This Old House,* where Solomon is building a house and fetched Hiram, who was *"filled with wisdom and understanding, and cunning. . . ",* sort of a biblical Norm Abrams. Hiram, among other things, *"made a molten sea,* ***ten cubits*** *from one brim to the other: it was round all about . . . and a line of* ***thirty cubits*** *did compass it round about."*

I suspect that those dimensions approximate. If they are accurate, then the outline must have been flattened, like some neolithic stone circles.

Mandala

Slide 9-112: Tibetan *Mandala of Yamaktaka,* from Arguelles cover

The most beautiful examples of squaring the circle can be seen in the Indian or Tibetan **mandala**. In Sanskrit, Mandala literally means *circle and center* or *Holy Circle*, and is essentially a vehicle for concentrating the mind.

Note that a Mandala often contains a square as well as a circle, and even if the square does not "square" the circle the symbolism is still there.

Carl Jung says that the circle symbolizes the processes of nature or of the cosmos as a whole; the square refers to the universe as conceived and projected by man.

Summary

Slide 9-124: Compass and Square Dust jacket detail
Kemp, Martin, *The Science of Art.* New Haven: Yale U. Press, 1990.

We've seen several recurring themes in this unit, and in preceding ones. They are:

- Reconciliation of Opposites
- Transcending duality.
- Finding a Middle Ground Between Extremes
- Mediating Between the Earthly and Divine

Specific symbols that we've seen relating to these themes are:

Vesica	Shows the duality of two circles, and also common ground.
Golden ratio	Represents the golden mean between two extremes.
Pyramids	Square the circle and reconcile triangle and square.

Vitruvian Man	Man inside a square and also a circle.
Flattened circle	An attempt to rationalize the irrational.
Mandala	Squares the circle.
Star of David	One triangle pointing to heaven intertwined with another pointing to earth.
Sri Yantra	Some triangles point upwards to the heavenly, others downward to the earthly.
Chrismon	A cross within a circle, uniting earthly and divine.
Compass & Square	Represents the squaring of the circle by means of the instruments used.
Orthogonal vesicas	Square the circle
Yin-Yang Symbol	Intertwined dark and light figures, reconciled by outer ring.
Rainbow	Connects heaven and earth

All of these can be seen as a geometric symbols of mankind's attempt to get in good with the cosmic forces that control our lives, and that's what religion tries to do.

Reading

Cowan, pp. 80-125

Lawlor pp. 31-35

Jung, *Man and His Symbols*, pp. 240-249.

Janson. The Chapter on *Gothic Art*

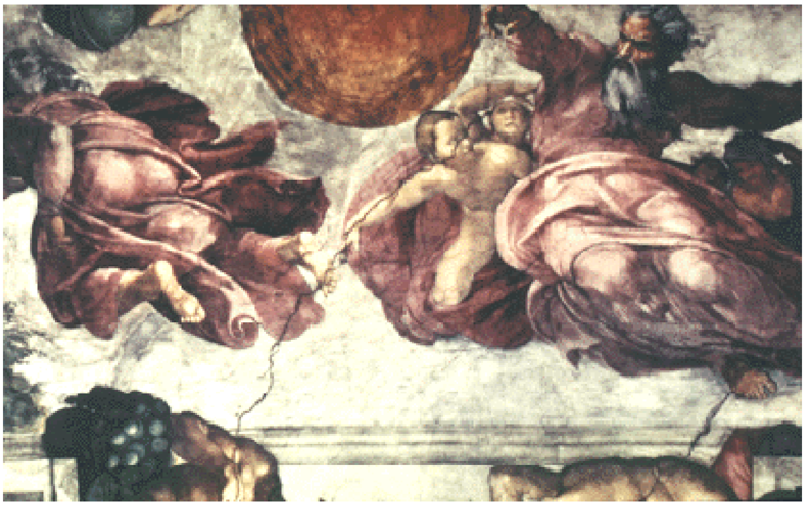
Eco

Arguelles

Celestial Themes in Art & Architecture

"When he established the heavens I was there: when he set a compass upon the face of the deep:"

Proverbs, Chapter 8 par. 27



Slide 4-38: MICHELANGELO: Creation of the Sun Sistine Chapel Ceiling.

Canaday, John. Masterpieces by Michelangelo. NY: Crown, 1979. p. 17

We will start on earth and travel upwards through the nine concentric spheres of the Ptolemaic system. We will follow the path of Dante and Beatrice in the *Paradiso,* looking for art motifs as we go.

The Ptolemaic system gave a geometric structure to the *celestial* world. In a later unit we'll see how linear perspective gave geometric structure to the *terrestrial* world.

The Ptolemaic Universe

God The Geometer

Slide 9-37: ***God the Geometer,*** **Manuscript illustration.**

Clark, p. 52

We vowed to search heaven and earth for geometric art motifs. We'll done a pretty good job on earth, so now lets try the heavens, and what better place to start than with God creating the universe. It almost seems like Medieval man thought he laid it out with a pair of compasses, a notion which may be due to a passage from the Old Testament:

"When he established the heavens I was there:

when he set a compass upon the face of the deep:"

The Four Realms

Lets look at a Medieval version of the universe, consisting of the Four Realms that we mentioned in our unit on number symbolism. Now lets group them into two sets of two;

Two *below* the moon, or *sublunary*: Matter
Nature
and two *above* the moon, or *Translunary*: Celestial:
Super-celestial

The Nine Spheres of Heaven

Slide 10-2: Giovanni di Paolo, *Creation of the World & Expulsion from Paradise*,1445

Fisher, Sally. The Square Halo. NY: Abrams, 1995. p. 14

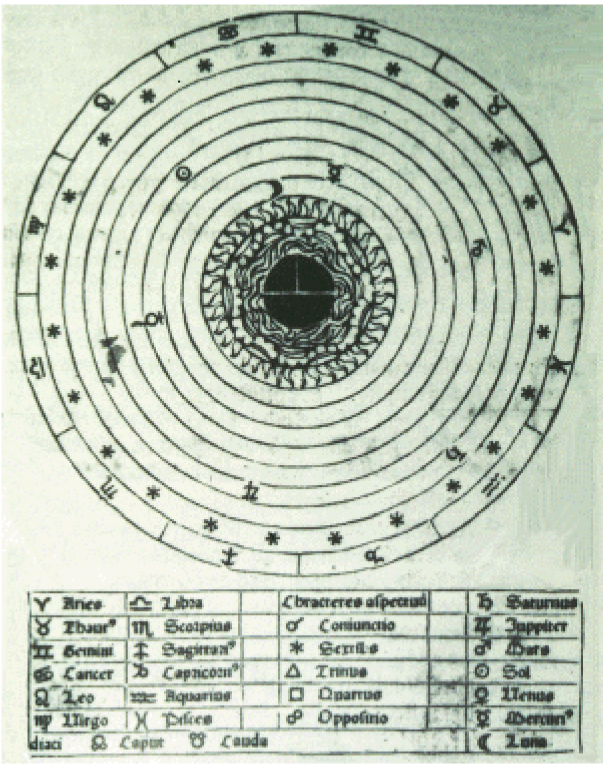
We now further divide the celestial realm into the 9 spheres of the heavens, the sun, moon, 5 planets, the fixed stars, and the primum mobile, and we get the geocentric universe shown in this picture by Giovanni di Paolo. It shows:

The **Earth** (brown) at center, shown as a *Mappamondo,* or world map, surrounded by the three other elements, **water, air, and fire**, its bright red clearly marks the boundary between the sublunary and translunary realms. Then comes the **moon and planets**, all blue, except for the Sun, yellow-white, with gilded sunburst, and Mars, in pink, for the red planet. Then the **fixed stars** with signs of the zodiac, the ***Primum mobile*** (the first moved) which regulated the motion of all the spheres beneath it, and the **Empyrean heaven**, the home of God and the angels.

The number of rings in pictures of this sort vary from one to the other. For one thing, theologians couldn't decide whether the empyrium occupied a definite sphere, or whether it was infinite and unknowable - a big problem for artists.

Di Paolo shows no ring for the Empyrium, just a region beyond the last ring, implying it can't be contained by a circular boundary.

Sacrobosco's *Sphaera mundi*:



Slide 10-7: Illustration from Sacrobosco

Dixon, Laurinda S. Giovanni di Paolo's Cosmology. Art Bull. Dec. 1985, p. 604-613.

The probable source for di Paolo's picture and others like it was Sacrobosco's *Sphaera mundi*: a popular source for this information written in the 13th century and used at universities. It presented an elementary and introductory view of the universe, giving Greek cosmology with a Christian spin.

So we have what Edgerton calls the *Geometrization of Heavenly Space*, the counterpart of our geometritization of terrestrial space acheived with linear perspective, where all receding lines travelled obediently to a neat vanishing point at infinity. This is part of a world view ruled by a general conception of an orderly universe that God had created out of chaos, where, according to the Book of Wisdom, *God had arranged all things according to Number, Weight, and Measure.*

The Sublunary World

Lets take the *Sphaera mundi* as our road map for our journey, starting with a quick look at the sublunary realm before taking off for the cosmos.

Sublunary vs. Superlunary

Compared to the rest of the Medieval universe, our sublunary realm is not so great. We may think that earth being at the center of the is special and exalted, but things are just the opposite.

Listen to Tillyard;

. . . far from being dignified . . . the earth in the Ptolemaic system was the cesspool of the universe, the repository of its grossest dregs.. .

By the *grossest dregs* he probably means people.

Portrayal of Astrologers and Astronomers

Slide 10-101:

Bouleau, Charles. The Painter's Secret Geometry. NY: Harcourt, 1963. p. 78

Well, maybe we're the dregs of the universe, but we can still gaze at the heavens and wonder, and the astrologers and astronomers who do that have always been a popular art motif, as those in this medieval manuscript illustration.

Allegories to Astronomy

Slide 10-104: RAPHAEL, *Astronomy*

Edgerton, Samuel. The Heritage of Giotto's Geometry. p. 104

Artists also like to depict allegories to astonomy, which, we recall, is one of the Seven Liberal Arts, part of the *quadrivium.*

Models of the Heavens

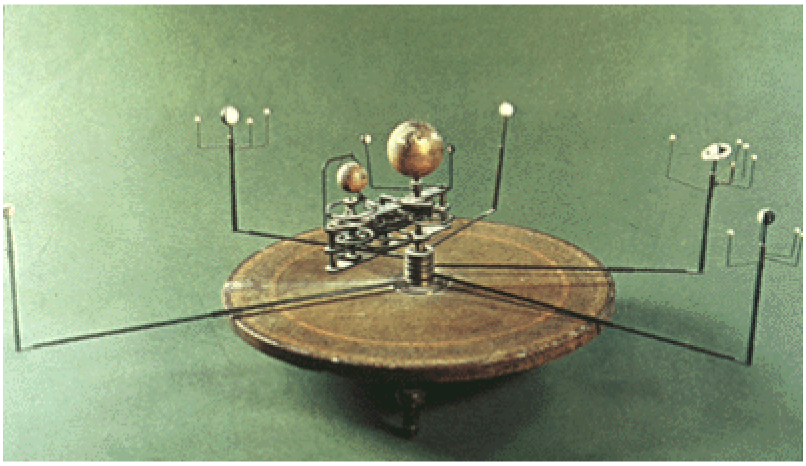
Slide 10-112: Villa Farnesina

Art Bulletin, September '95, p. 421

Slide 10-114: Coronelli (1650-1718) Globes, c. 1688.

Museo Civico, Vicenza

Here in the sublunary world we make pictures and models of the translunary world, like star maps, often painted on ceilings, celestial globes, and orreries or planiteria, little models of the solar system.



Slide 10-117: Orrery

Turner, Gerard. Antique Scientific Instruments. Dorset: Blandford, 1980. Fig. 10

Armillary Sphere

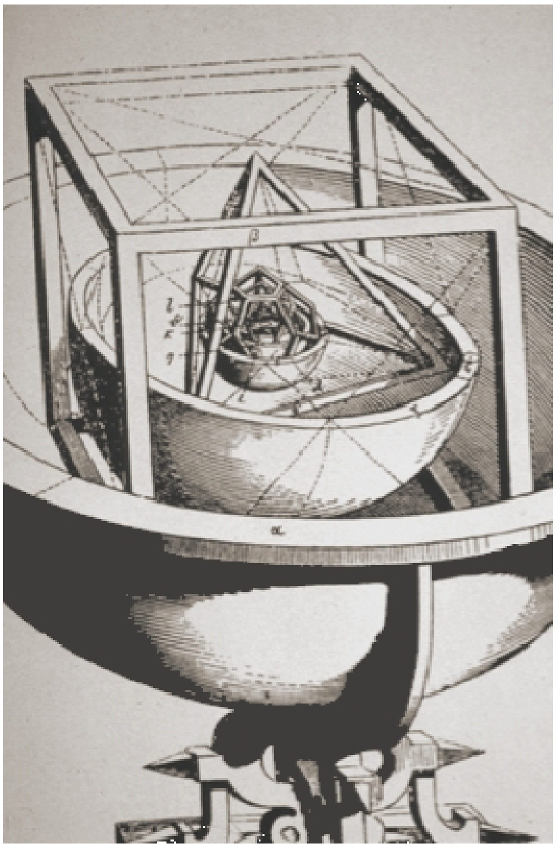
Slide 10-121: Armillary Sphere

Turner, Gerard. Antique Scientific Instruments. Dorset: Blandford, 1980. p. 61

And of course, the armillary sphere, which we saw before in Plato's *Timaeus*, where he describes how the circular paths for the stars were formed by the creator.

"He cut the whole fabric into two strips,which he placed crosswise at their middle points to form a shape like the letter X; he then bent the ends round in a circle and fastened them to each other . . . to make two circles, one inner and one outer."

The Dome of Heaven



Slide 3-6: Kepler's Model of the Universe

Lawlor, p. 106

Another model of the heavens is that we've seen before is Kepler's nested Platonic solids, and another is the dome. In *The Dome of Heaven,* Karl Lehmann, who writes,

One of the most fundamental artistic expressions of Christian thought and emotion is the vision of heaven depicted in painting or mosaic on domes . . .

Instruments

Slide 10-108:Astrolabe

Brenni, Paolo et al. Orologi e Strumenti della Collezione Beltrame. Florence: Instituto e Museo di Storia della Scienza, 1996. Fig. I

Many instruments used by astronomers, often beautifully made and ornamented, easily qualify as art objects, from this small astrolabe to these large installations in India.

Slide 10-107: Astronomical Sites in India

Sharma. L'Observatoire Astronomique de la Ville Rose

Sundials

Slide 10-127: Sundial at Chartres

Lionass, p. 28

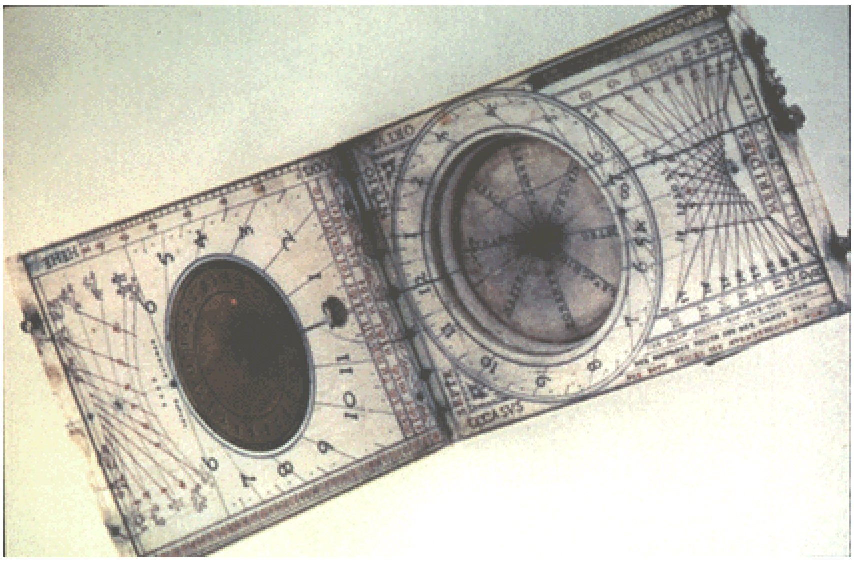
Many such instruments use shadows or shafts of light to mark the passage of time, like sundials or sun clocks, that came in all sizes, from ones you can fit into your pocket, to dials mounted on buildings.

Slide 10-133: Sundials

Lionass, Francois. Time. NY: Orion, 1959. p. 52

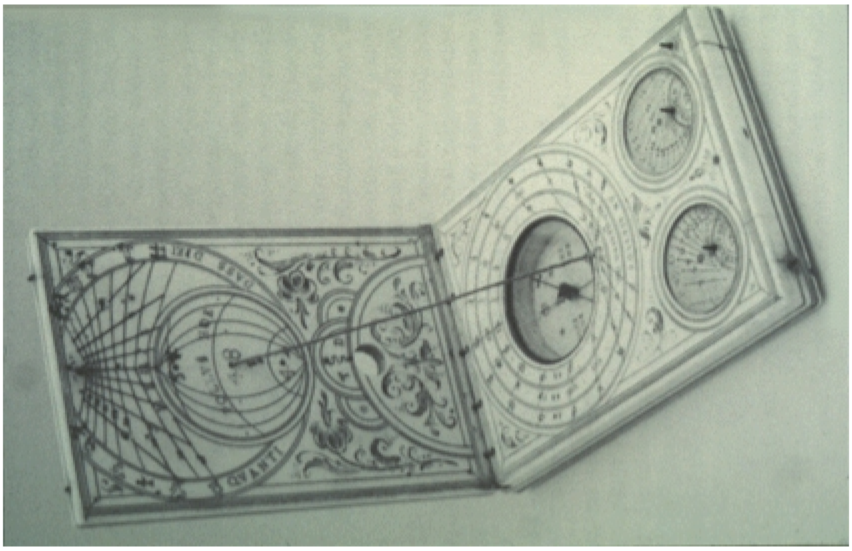
Slide 10-134: Sundials

Lionass, Francois. Time. NY: Orion, 1959. p. 69



Slide 10-135:

Brenni, Paolo et al. Orologi e Strumenti della Collezione Beltrame. Florence: Instituto e Museo di Storia della Scienza, 1996. Figure III



Slide 10-136: Sundial

Brenni, Paolo et al. Orologi e Strumenti della Collezione Beltrame. Florence: Instituto e Museo di Storia della Scienza, 1996. p. 51

Slide 10-137: Sundial

Brenni, Paolo et al. Orologi e Strumenti della Collezione Beltrame. Florence: Instituto e Museo di Storia della Scienza, 1996. p. 41

Slide 10-138: Sundial

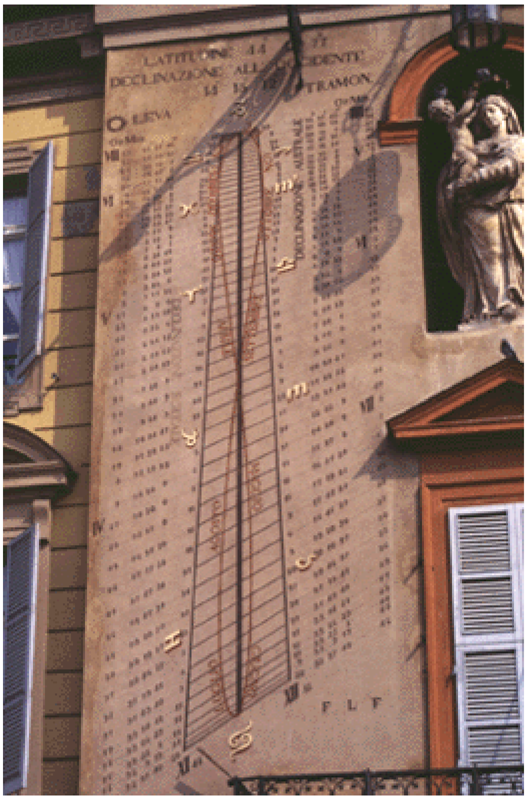
Turner, Gerard. Antique Scientific Instruments. Dorset: Blandford, 1980. Figure 8

Sun Calendars

Slide 10-140: Sun Dagger

Lippard, Lucy. Overlay. NY: Pantheon, 1983. p. 76

Some calendars only mark particular times of the year; solstices, equinoxes, like this sun dagger in New Mexico that marks the summer soltice, or the calendar circles like Stonehenge and Castle Rigg.



Slide 10-141: Parma Town Hall

Calter Photo

Other sun calendars give the approximate date by seeing where the noon mark falls on a figure called the Analemma

Slide 22-01: Sun Disk analemma

citation

Another sun calendar is located in S. Petronio, Bologna, where a hole in the ceiling of the cathedral projects a shaft of sunlight onto this bronze strip on the pavement below which is engraved with the days of the year and signs of the zodiac.

Slide 10-143: Meridan Line. S. Petronio, Bologna

Calter Photo

A more modern and more grim sun calendar is this Kentucky Vietnam Veterans Memorial, where the tip of the shadow of the rod falls on the names of those who died in the Vietnam war at the current date and time of day.

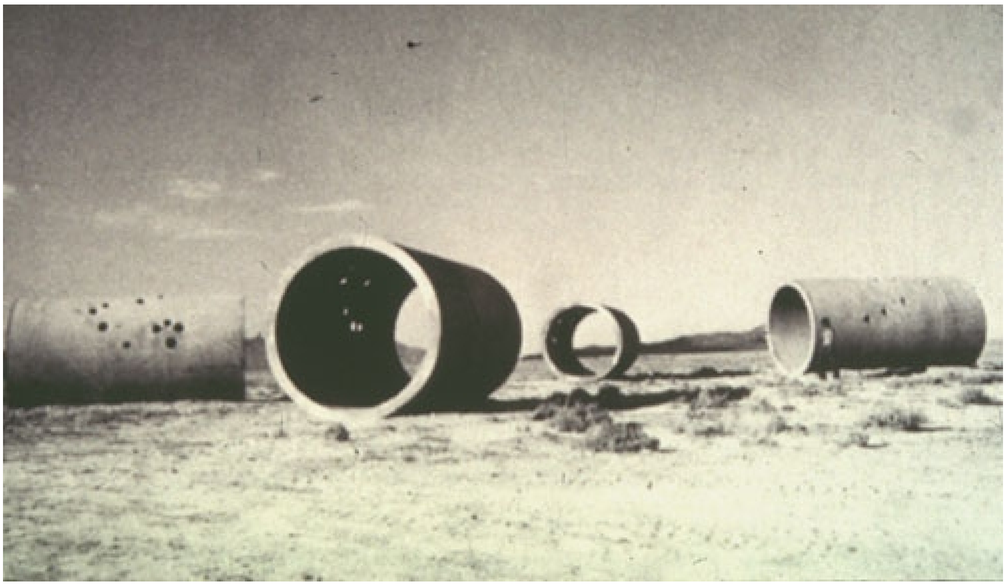
Slide 10-153: Kentucky Vietnam Veterans Memorial

Calter Photo

Slide 22-02: Nancy Holt, Annual Ring, 1980-91
Lippard, Lucy. Overlay. NY: Pantheon, 1983. p. 107

Slide 22-03: Robert Morris, Observatory, 1970-77
Lippard, Lucy. Overlay. NY: Pantheon, 1983. p. 110

Some twentieth century art use alignments, like Nancy Holt's *Annual Ring*, with openings that frame the rising and setting sun on the equinoxes, and the north star. Another is Robert Morris', Observatory, nearly 300 ft in diameter, which has slits for the solstices, equinoxes, and moonrise.



Slide 22-04: Nancy Holt, Sun Tunnels, 1973-76
Lippard, Lucy. Overlay. NY: Pantheon, 1983. p. 107

Nancy Holt's *Sun Tunnels* are oriented to the solstices and the holes project certain constellations on the interior dark walls, and Calter's sculptures sometimes function as working sundials or sun calendars.

Slide 22-05: Armillary VII

Calter Photo

Slide 22-06: Sun Disk, Moon Disk
Calter Photo

The Nine Circles of Heaven

But enough of this murky sublunary world where the air thick and dirty. Lets have a space odessy to the translunary world where the ether is clear and pure, to the celestial and the super-celestial spheres, the nine circles of heaven.

Slide 10-15: DI PAOLO: ***Dante leaving the Earth***

Pope-Hennessy, John. Paradiso. The Illuminations of Dante's Divine Comedy by Giovanni di Paolo. NY: Random, 1993. p. 74

On our trip lets follow in the footsteps of Dante and Beatrice, in his *Paradiso,* shown here leaving the earth for the moon.

Dante's *Paradiso*

Paradiso is one book of the *Divine Comedy* written by the poet Dante Alighieri (1265-1321). It was started about 1307 was completed shortly before his death. It is an allegorical narrative of the poet's imaginary journey through hell and purgatory, guided by Virgil, who is, to Dante, the symbol of reason.

Slide 10-13: DELACROIX: *Dante and Virgil*

Clark, Kenneth, *The Romantic Rebellion.* NY. Harper, 1972. p. 202

Dante is guided through the circles of heaven by Beatrice, a woman he met in 1274, and whom he loved and exalted in *La vita nuova* (The New Life) and in *Paradiso.*

Slide 10-12: ***Dante and Beatrice***

Dante, The Divine Comedy. Ill. Gustave Doré. London: Cassell. p. 183

In each realm the poet meets mythological, historical, and contemporary personages, each symboliizing a particular fault or virtue, each receiving the appropriate punishment or reward. In his *Paradiso* he conceived of heaven as a gigantic rose, built of circular rings of light simialr to the rose window of a Gothic cathedral.

Nine Ranks of Angels

Slide 10-8: HILDEGARDE VON BINGEN. ***Nine Ranks of Angels***

Fox, Matthew, Illuminations of Hildegard of Bingen. Santa Fe: Bear, c 1985. p. 74

When we left the world of people we entered the realm of the angels, supposedly arranged in three ranks of three -- a triple trinity -- shown here in Hildegard von Bingen's Nine Ranks of Angels.

Lowest Heirarchy:

Angels	**Moon**
Archangels	**Mercury**
Principalities	**Venus**

Middle Heirarchy:

Powers	**Sun**
Virtues	**Mars**
Dominations	**Jupiter**

Upper Heirarchy:

Thrones	**Saturn**
Cherubim	**Fixed Stars**
Seraphin	**Emperian**

This arrangement is from a sixth-century book, *The Celestial Hierarchy,* ascribed to the neoPlatonist known as the *Pseudo-Dionysius.* (This is not the fun Dionysius of Greek Mythology, the god of wine, orgies, Phallicism, and dorm parties.)

Circle 1. The Moon

The Illustrations for *Paradiso*

We'll look at two sets of illustrations for *Paradiso*; the ones in color are by the Sienese artist Giovanni di Paolo, done about 1445, and the monochrome etchings are by Gustav Doré, done in the 1900's. Occasionally we'll have illustrations of the same passage by the two artists, like these, where Dante and Beatrice converse with Piccarda dei Donati and the Empress Constanza, who broke their vows and thus occupy the lowest sphere of heaven.

Slide 10-17: ***Donati and Costanza***

Dante, The Divine Comedy. Ill. Gustave Doré. London: Cassell. p. 182

Slide 10-16: DI PAOLO: ***Donati and Costanza***

Pope-Hennessy, John. Paradiso. The Illuminations of Dante's Divine Comedy by Giovanni di Paolo. NY: Random, 1993. p. 76

We'll see that the di Paolo illustrations are generally more specific and more literal than Doré's. They refer to a specific passage in the allegory, while the Doré pictures can usually go anywhere in the story.

Cigoli's *Immacolata*

Slide 10-27: TIEPOLO: *Immaculate Conception,* 1767.

Fisher, Sally. The Square Halo. NY: Abrams, 1995. p. 53

A popular art motif featuring the moon is the Virgin standing on the moon, usually wearing a crown of stars.

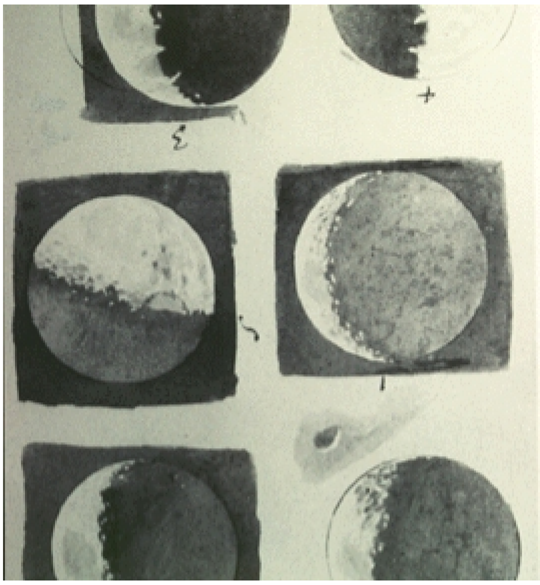
And a great portent appeared in heaven, a woman clothed with the sun, with the moon under her feet, and on her head a crown of stars.

Notice the radiance coming from her body, showing her *clothed in the sun.*

Slide 10-25: CIGOLI' *Immocolata*

Edgerton, Samuel. The Heritage of Giotto's Geometry. p. 252

But we're particularly interested in this version, done in 1612 by Cigoli. Note the mandorla, the nine celestial circles, and the dome as a model of the heavens. But especially look at the moon. For the first time it is shown *pock-marked.*



Slide 10-26: Galileo's Moon Drawings

Edgerton, Samuel. The Heritage of Giotto's Geometry. p. 241

Why? It turns out that Cigoli was pals with Galileo. Quoting Panofsky,

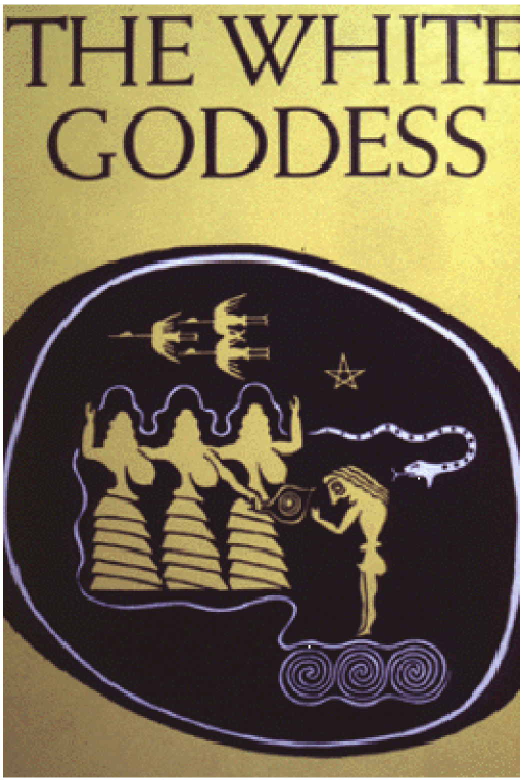
". . . the painter, as a good and loyal friend [to Galileo] paid tribute to the great scientist by representing the moon under the virgin's feet exactly as it had revealed itself to Galleo's telescope -- complete with . . . those little . . . craters which did so much to prove that the celestial bodies did not essentially differ . . . from our earth.

This may not seem like a big deal now but it defied convention and church doctrine that showed the moon either as a crescent or as a perfectly smooth orb, perfect and flawless as the person standing on it.

Moon Gods & Symbols

Slide 10-18: CORREGIO: ***Diana***

Cayley, p. 63



Slide 4-8: The White Goddess

Graves Cover

Of course, the moon had associations long before Christianity, like the moon goddesses Isis and Selene, and the triple goddess of the New, Full, and Old Moon, goddess of Birth, Love, and Death.

Angels

Slide 10-11: DELACROIX: ***Jacob Wrestling with an Angel***

Clark, Kenneth, *The Romantic Rebellion.* NY. Harper, 1972. p. 220

Note that the moon has the lowest kinds of angels who do the grunt work, TV series, sell cellular phones, and wrestling with earthlings like Jacob.

Circle 2. Mercury

Slide 10-35: ***Mercury***

Pope-Hennessy, John. Paradiso. The Illuminations of Dante's Divine Comedy by Giovanni di Paolo. NY: Random, 1993. p. 87

Lets now head for Mercury, shown by di Paolo as a golden disk. Mercury, naked, stands surrounded by seven heavenly intelligences. Humanity, represented by the four youths, are being guided by both the Old and New Testaments.

Archangels

Mercury is the sphere that has the most interesting rank of angels, the *archangels.* There are only three, *Raphael, Gabriel,* and *Michael.* Raphael appears in the Old Testament Apocrypha *Tobit*, in some tale involving a fish, as in this drawing by Rembrandt.

Slide 10-37: REMBRANDT: *Tobias and Raphael*

Ward, Roger. Durer to Matisse - Exhibition Catalog. Kansas City: Nelson, 1996. p. 101

Gabriel is the archangel of the Annunciation, shown in this painting by Fra Angelico, one of the best portrayers of the annunciation.

Slide 10-39: FRA ANGELICO: *Annunciation*

Hartt, Frederic. Italian Renaissance Art. NY: Abrams, 1994. p. 14

But the superstar of the archangels is Michael from *Revelations.* Michael is often shown banishing Lucifer and the rebellious angels to hell.

Slide 10-41: BLAKE: ***Michael***

Clark. Romantic Rebellion, p. 156

Circle 3. Venus

Slide 10-35: ***Venus with Cupid and Amor***

Pope-Hennessy, John. Paradiso. The Illuminations of Dante's Divine Comedy by Giovanni di Paolo. NY: Random, 1993. p. 97

On to Venus, where di Paolo shows her with two sons Cupid and Amor.

Slide 10-36: ***Talking to Charles Martel***

Dante, The Divine Comedy. Ill. Gustave Doré. London: Cassell. p. 208

Slide 10-37: DI PAOLO: ***Talking to Charles Martel***

Pope-Hennessy, John. Paradiso. The Illuminations of Dante's Divine Comedy by Giovanni di Paolo. NY: Random, 1993. p. 98

Here we have another direct comparison of pictures by the two artists, showing one Charles Martel who is telling Dante the story about how he lost Sicily.

Venus Coelestis & Venus Vulgaris

Slide 10-38: BOTTICELLI: ***Birth of Venus***
Slide # 1055

American Library Color Slide Company p. 187

There are *actually two* Venuses, discussed in Plato's *Symposium,* Celestial Venus and Earthly Venus. The two Venuses correspond to the notion of *Sacred and Profane love,* a big topic in the Renaissance, shown here in a painting by Titian.

Slide 10-39: TITIAN: *Sacred and Profane Love*

Christiansen, Keith. Italian Paintings. NY: Levin Assoc. 1992. p. 194

Circle 4. The Sun

Slide 10-42: DI PAOLO:

Pope-Hennessy, John. Paradiso. The Illuminations of Dante's Divine Comedy by Giovanni di Paolo. NY: Random, 1993. p. 102

Here Dante and Beatrice reach the sun, shown by di Paolo as a golden wheel sending golden rays to the landscape below. The Sun, located in the middle of the orbs, with three lesser above and three below, like the heart in the middle of the body, or a wise king in the middle of his kingdom.

Sun Gods & Symbols

Slide 10-44: Aztec Sunstone
Dia.: 11' 2", wt.: 24 tons

Argüelles, José and Miriam. Mandala. Boston: Shambhala, 1985. p. 37

Recall that the circle was often used to symbolize the sun, and that Sun worship is one of the most primitive forms of religion, with early man often distinguishing between the triad of rising, midday, and setting sun, like the three Egyptian sun gods; Horus, the rising sun; Ra or Rê, themidday sun; Osiris, the old setting sun.

Circle 5. Mars

Slide 10-47: ***Cross***

Dante, The Divine Comedy. Ill. Gustave Doré. London: Cassell. p. 242

Slide 10-48: DI PAOLO: ***Cross***

Pope-Hennessy, John. Paradiso. The Illuminations of Dante's Divine Comedy by Giovanni di Paolo. NY: Random, 1993. p. 126

Lets zip by Mars, the red Planet, shown in that color by di Paolo. Here we see two different representations of the cross made up of eight holy warriors, Joshua, Judas Maccabaeus, Charlemagne, and more.

Circle 6. Jupiter

Slide 10-53: Eagle

Dante, The Divine Comedy. Ill. Gustave Doré. London: Cassell. p. 266

Slide 10-54: DI PAOLO: *Eagle*

Pope-Hennessy, John. Paradiso. The Illuminations of Dante's Divine Comedy by Giovanni di Paolo. NY: Random, 1993. p. 131

For Jupiter we have another good comparison of the illustrations by Paolo and Doré. Here the souls of the Just Rulers form into an eagle symbolizing divine justice and imperial authority, and speak with a single voice from its beak.

Circle 7. Saturn

Slide 10-57: DI PAOLO:*Saturn*

Pope-Hennessy, John. Paradiso. The Illuminations of Dante's Divine Comedy by Giovanni di Paolo. NY: Random, 1993. p. 137

Slide 10-56: GOYA: *Cronos devouring Children*

citation

On to Saturn, shown as an old man with a sickle. Saturn often represented *father time* because of the confusion between Chronos, the Greek word for time, and Kronos, the Roman for Saturn.

The sickle may represent the grim reaper, eventually mowing down every living thing, or the instrument he used to castrate his father Uranus. Saturn is often shown devouring his children, signifying that "sharp-toothed" time devours whatever he has created.

Circle 8. The Fixed Stars

Slide 10-58: DI PAOLO: *Stars*

Pope-Hennessy, John. Paradiso. The Illuminations of Dante's Divine Comedy by Giovanni di Paolo. NY: Random, 1993. p. 145

As they rise to the fixed stars, Dante and Beatrice look back and see the seven planets beneath them; the sun, lower left, a scarlet figure in a flaming chariot, to the right the moon, Mars, Mercury, Jupiter, Venus, and Saturn, arranged by the days of the week.

Slide 10-62: *St. John Questions Dante*

Dante, The Divine Comedy. Ill. Gustave Doré. London: Cassell. p. 300

Slide 10-63: DI PAOLO: *St. John Questions Dante*

Pope-Hennessy, John. Paradiso. The Illuminations of Dante's Divine Comedy by Giovanni di Paolo. NY: Random, 1993. p. 158

Here's another direct comparison between Dore and di Paolo, where St. John questions Dante on the subject of Charity, one of the three ecclesiastical virtues.

Cherubim

Slide 10-64: Cherubim. S. Marco Cupola

Demus, Otto. The Mosaic Decoration of San Marco, Venice. Chicago: U. Chicago, 1988. P. 44A.

We haven't seen any angels since we left the archangels. Each sphere has its own type, but they have not been depicted by artists. But here in the fixed stars we have the *Cherubim,* little babylike creatures. They are probably recycled classical *putti* .

Sometimes only winged heads are shown. Others are shown with several sets of wings, often in the shape of a cross.

The Zodiac

Slide 10-67: Fountain of Neptune, P. della Signoria, Florence

Calter Photo

Slide 10-68: Torah Crown detail, c. 1770

Jewish Museum (New York, N.Y.), Treasures of the Jewish Museum. NY: Universe, 1986. p. 103

Stars as art motifs are found mostly in the zodiac, and we find these everywhere.

The Stars in Painting

Slide 10-69: TINTORETTO: *Origin of the Milky Way*, c. 1577

Kent, p. 21

The star-filled sky has fascinated artists. Tintoretto's painting shows a rather literal explanation for the *Origin of the Milky Way*. Edvard Munch did two *Starry Nights*, and, of course, Van Gogh made another painting with the same title.

Slide 10-74: MUNCH:*Starry Night*, 1893. (At Getty)

Munch, p. 106

Circle 9. The Primum Mobile

Slide 10-75: Primum Mobile

Dante, The Divine Comedy. Ill. Gustave Doré. London: Cassell. p. 310

Slide 10-76: DI PAOLO: Primum Mobile

Pope-Hennessy, John. Paradiso. The Illuminations of Dante's Divine Comedy by Giovanni di Paolo. NY: Random, 1993. p. 162

We now come to the last sphere, the primum mobile, or *first moved,* the sphere which dictates the motions of the other spheres. Here it is shown by di Paolo as a ring of golden light framing a *mappamondo* in the center of which is the figure of Christ.

Slide 10-79: MANTEGNA: *Assention*, showing seraphim

Campbell, Joseph, with Bill Moyers. *The Power of Myth.* NY: Doubleday 1988. figure 19

The angels assigned to this sphere by Dionysius are the *seraphim,* usually depicted like cherubim, but are red.

The Empyrium

Slide 10-82: DI PAOLO: *Cosmic Rose*

Pope-Hennessy, John. Paradiso. The Illuminations of Dante's Divine Comedy by Giovanni di Paolo. NY: Random, 1993. p. 177

We finally reach the Empyrian, the highest heavenly realm, supposed to be composed by a kind of sublimated fire, the uppermost Paradise, the heaven; the seat of God. The image that dominates the final Cantos of *Paradiso* is the *cosmic rose,* shown by di Paolo as an actual rose, with nine angels and the Trinity.

And in the very last paragraph of *The Divine Comedy,* at the end of this fantastic journey down to hell and back, and through purgatory, and up through the circles of heaven, what does Dante talk about? Beatrice? God? No. He talks about*geometry.*

As the geometer who attempts to measure the circle
and discovers not . . . the principle he wants,
So was I at that new sight

I wished to see how the image conformed to the circle
[but] here my power failed,
but my desire and my will were revolved,
like a wheel that is evenly moved
by the love which moves the sun and the other stars.

So we end our journey to the heavens with love and with geometry; what more could anyone who loves math ask for?

Reading

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Janet Saad-Cook. *Natural Phenomena, Earth, Sky, and Connections to Astronomy.* Leonardo, V.21, No. 2, 1988, pp. 123-134

Vitruvius, Book IX. Dover Edition pp. 251-277.

Panofsky, *Studies in Iconology*, pp. 129-169

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Pacholczyk, Josef. *Music and Astronomy in the Muslim World.* Leonardo, V29, No. 2, pp. 145-150, 1996

Tillyard, *The Elizabethan World Picture.* pp. 37-82

Ostrow, Steven. *Cigoli's Immacolata and Galileo's Moon.* Art Bull. June '96, pp. 218-235.

Dante, *Paradiso*

Pope-Hennessey, *Paradiso*

Pseudo-Dionysius, Smith

BRUNELLESCHI'S PEEPSHOW
&
THE ORIGINS OF PERSPECTIVE

"Perspective is the rein and rudder of painting"

Leonardo da Vinci

Slide 12-5: Aerial View of Santa Maria della Fiore, the Duomo, and the Bapistry.

Edizioni Becocci-Firenze

We move now into the early Renaissance, with a survey of linear perspective.

The entire theory of perspective can be developed from a single fact: that *the apparent size of an object decreases with increasing distance from the eye.* That's the phenomenon that makes railroad tracks appear to converge in the distance.

We will start with explanations of what perspective is, and then outline its historical development, from Brunelleschi's experiment to Alberti's Treatise, in the early Renaissance, followed by early examples of the use of perspective. We will then quickly survey what has happened to perspective up to the present time.

What is Perspective?

Pictures before Perspective

Slide 12-1: Inital word panel of Psalm from the Kaufmann Haggadah. Spain, late 14th C. Keller, Sharon. *The Jews: A Treasury of Art and Literature.* NY: Levin Assoc. 1992.

These perspectival errors appear in paintings usually done before 1400.

The perspective lines usually converge, but not to a single point and not on the horizon.

Pictures after Perspective


Slide 3-1: RAPHAEL: ***School of Athens*** American Catalog, p. 126, #21061

Shows one-point (linear perspective)

Brunelleschi's Peepshow

Slide 12-4: BRUNELLESCHI:
Statue of Brunelleschi

Calter Photo

So what happened? Why did pictures change from those without perspective to those that had perspective? For one thing, Brunelleschi happened!

Filippo Di Ser Brunellesco (1377-1446), sculptor, architect, and artisan-engineer, is given credit for the invention of linear perspective. Here he is looking up at the famous dome he built for the Duomo (1418-1436) in Florence. He also built San Lorenzo and many other structures.

Slide 12-6: Cathedral

Firenze, N-26

The Peepshow

Slide 12-10: The Duomo and Piazza del Duomo

FIRENZE, N-23

Imagine that it is sometime during the early 15th century, between 1410 and 1420, in this Piazza Duomo in Florence. You see Brunelleschi standing in the west door of the unfinished cathedral. He beckons eagerly to you and has you face the Baptistery across the piazza. This the same Baptistery we saw pictures of when we discussed the octogon.

Slide 12-11: The Bapistry, San Giovanni

FIRENZE, N-20

He holds up a picture of the Baptistery painted on a panel, its back towards you, and has you squint through a small hole in the painting.

According to Brunelleschi's biographer Antonio Manetti, Brunelleschi *"had made a hole in the panel . . . which was as small as a lentil on the painting side . . . and on the back it opened pyramidally, like a woman's straw hat, to the size of a ducat or a little more."*

Through the hole you saw a mirror which reflected the painting itself so you see the front of the painting in the mirror. Then Pippio whisks away the mirror so that you see the real Baptistery through the peephole and you are amazed because they are so similar.

But this was no ordinary painting. It is said to have been the first accurate perspective picture.

No written record exists from Brunelleschi's experiments. He probably passed the method verbally to Masaccio, Masolino, and Donatello, who used it in their works.

Project: Reproduce the Peepshow demonstration.

Donatello

The earliest surviving use of linear perspective in art is attributed to Donato di Niccolò di Betto Bardi (1386-1466), called Donatello, who is considered by many to be the greatest sculptor of the early Renaissance in Italy, and perhaps one of the greatest sculptors of all time.

St. George and the Dragon

Slide 12-12: DONATELLO: ***Relief: St. George and the Dragon***

Calter Photo

Donatello later did some marble statues for the facade of the cathedral in Florence, and two for Orsanmichele, his *St. Mark* and his *St. George* (c. 1412). The latter is particularly interesting to us. Below the main sculpture-in-the round of Saint George is a marble relief showing him slaying the dragon.

In this panel, sometimes called a *rilievo schiacciato* (ski-a-chat-o) or flattened relief,

Donatello appears to be using linear perspective. However the orthogonals are not so clearly defined that one can say for certain that he is using perspective, or to let us locate a precise vanishing point.

The *Feast of Herod*

Slide 12-13: Bapistry Font, Siena

Slide 12-14: DONATELLO: *Feast of Herod,* c. 1425

Calter Photo

There is no lack of certainty, however, about Donatello's use of perspective in his bronze *Feast of Herod.* This panel, made for the font of the Siena cathedral, (which we saw before when discussing the octogon) is widely recognized as the first relief sculpture to use linear perspective.

The orthogonals are not very long or prominent, but what there is of them can be seen to intersect at a vanishing point near the elbow of the musician in the central window. The vanishing point would be at eye level for the figures seated behind the table, if they were sitting up straight.

It has been pointed out that Donatello's use of linear perspective is imperfect here, because the orthogonals do not quite meet in a single vanishing point, but this imperfection is small and may have occurred in the chasing of the bronze, and not in the original design.

At any rate, it is a far more advanced perspective construction than in his *St. George* relief. It is no longer empirical, but is apparently based on theory.

Masaccio

Masaccio (1401-c.1428) is considered by many to be one of the triumvirate, along with Brunelleschi and Donatello, who laid the foundations for the Renaissance in Italy. Masaccio's first known painting, the triptych *Madonna and Child with Saints* (c.1422) shows no sign of linear perspective. However, three other paintings, all done in the last four years of his short life, do show perspective, and are the works on which Masaccio's fame rests.

Slide 12-16: MASACCIO:*Trinity*

Calter Photo

Of the three, Masaccio's *Trinity*, painted for S. Maria Novella in Florence around 1427, is usually considered to be the oldest surviving perspective painting. It is considered his most mature work.

We see a pyramid of figures topped by God, who holds the cross. The holy spirit is represented by the dove. St. John the Baptist on the right and the Madonna is on the left. The Madonna looks directly at us and presents Christ to us. The figures frame Christ, in the center.

Massacio is said to have consulted with Brunelleschi on this painting. He used a grid framework, tooled right into the surface, and very rigorous linear perspective. Even the nails are shown in perspective! He placed the vanishing point at the *Mound of Golgotha*, or Cavalry, the place outside Jerusalem's walls where Jesus was crucified, at the eye level of an average person. He probably felt that the illusion of depth would be greatest with the viewer right on the centric ray.

Perspective used to provide narrative focus:

Slide 12-18: MASACCIO: *Tribute Money,*
Janson, H.W. History of Art. Fifth Edition. NY: Abrams, 1995.

So we see that in addition to spatial organization and illusion of depth and a structural focus, perspective can provide narrative focus. Since the eye invariably travels to the vanishing point of a picture, Renaissance artists did not hesitate to put something important at or near that point. Masaccio's use of perspective is clearest in his frescos for the Brancacci Chapel in S. Maria del Carmine in Florence, which illustrate scenes from the lives of St. Peter and St. Paul. In particular, his *Tribute Money* (c. 1425) shows the confrontation of temporal and spiritual authority, and shows clear and effective use of perspective.

Perspective not only provides a visual structure for the painting, but a narrative focus, by having the vanishing point at Christ's head.

Project: Xerox a few paintings. Draw orthogonals and locate VP. Speculate as to why the artist placed it there.

Mantegna

Slide 12-19: MANTEGNA: ***Bust of Mantegna*** in Sant' Andrea, Mantua

Calter Photo

The artist-scholar Andrea Mantegna (active 1441, d.1506) used linear perspective with dramatic effect. Here we mention only his fresco cycle, largely destroyed by allied bombing during WWII, in the Ovetari Chapel of the Church of the Eremitani in Padua (1455), which depict scenes from the life of St. James.

Ovetari Chapel frescos

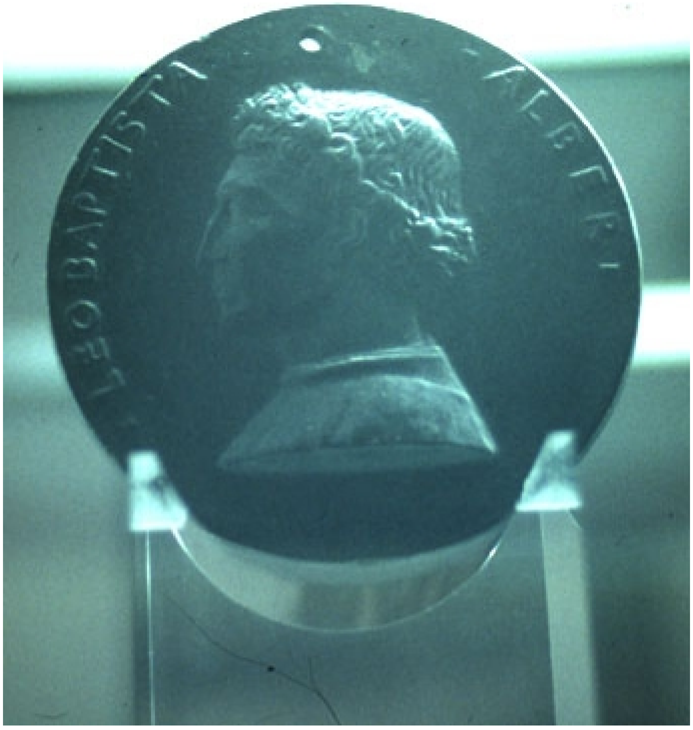
Slide 12-20: MANTEGNA: ***St. James before Herod Agrippa*****,c. 1455**

American Library Color Slides Co. Inc., Cat. #23354

The pavement in his *St. James before Herod Agrippa* is laid out in a strict grid, divided into cubits. The vanishing point in this painting is located in the frame *between* this painting and the one immediately to its left, and is shared by that painting.

The two lowest scenes in the cycle are made extremely dramatic by having a vanishing point *below* the bottom frame of the paintings. This placement may have been due to Donatello's influence.

Alberti



Slide 17-02 : ALBERTI:
Medal recto

Cole,p. 12 or Reti, p. 219

Leon Battista Alberti (1404-1474) was, according to the Encyclopedia of the Italian Renaissance,

"humanist scholar, natural scientist, mathematician, cryptographer, pioneer of the Italian vernacular, author." He was also a noted architect who built Sant' Andrea, were we just saw the bust of Mantegna, Santa Maria Novella in Florence, in which Masaccio's Trinity is painted, and many others.

Slide 17-03 : Sant' Andrea, Mantua

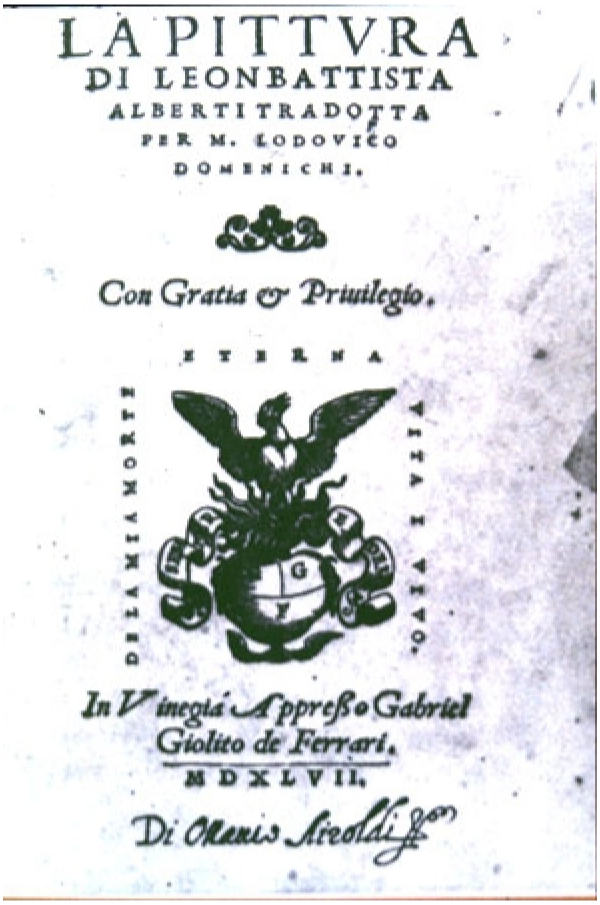
citation

Slide 17-04: Santa Maria Novella, Florence

He is said to have invented the *camera obscura*. Any camera, including the camera obscura, automatically makes perspective pictures. Thus if Alberti did indeed invent the camera, or was even familiar with the device, it is hard to imagine that this did not influence his development of perspective theory.

Project: Make a camera obscura

Della pittura


LA PITTVRA
DI LEONBATTISTA
ALBERTI TRADOTTA
PER M. LODOVICO
DOMENICHI.

Con Gratia & Priuilegio.

ETERNA

In Vinegia Appresso Gabriel
Giolito de Ferrari.
MDXLVII.

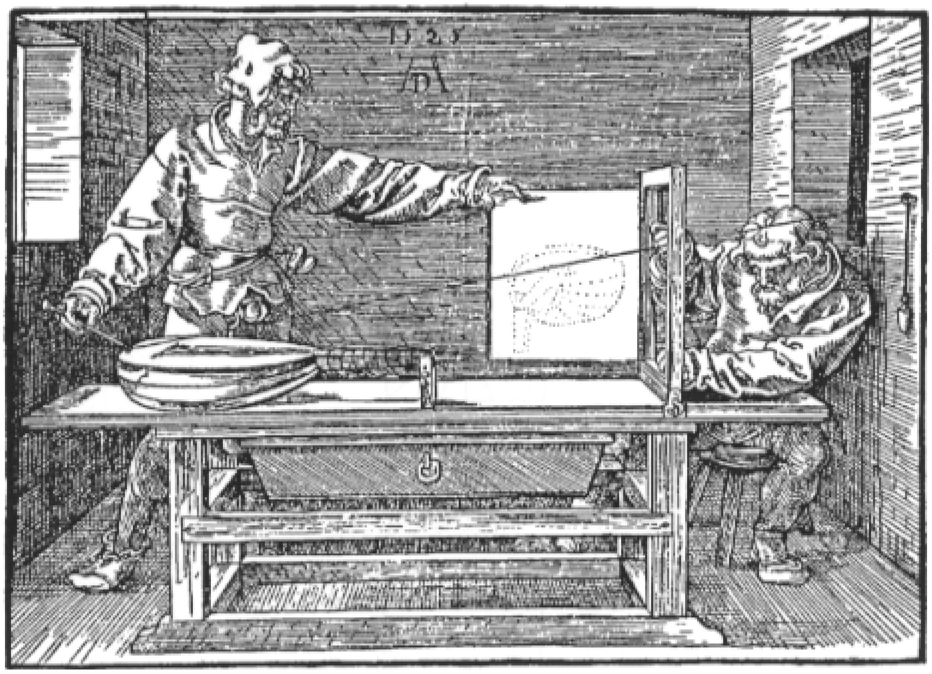
Slide 17-05: Cover of Della Pittura

citation

As Brunelleschi made no written record of his perspective findings, it remained for Alberti to be the first to put the theory into writing, in his treatise on painting, *Della pittura* (1435). There, Alberti gave practical information for painters and advice on how to paint *istoria* or history paintings. We will discuss only the section which gives the first written account of linear perspective.

The *Velo*

Alberti described how an artist could get a correct view of a scene by observing it through a thin veil, or *velo.* The idea is that we can get a correct image of some object seen through such a veil or a window by tracing the outline of the object on the window glass. Albrech Durer designed several such machines.

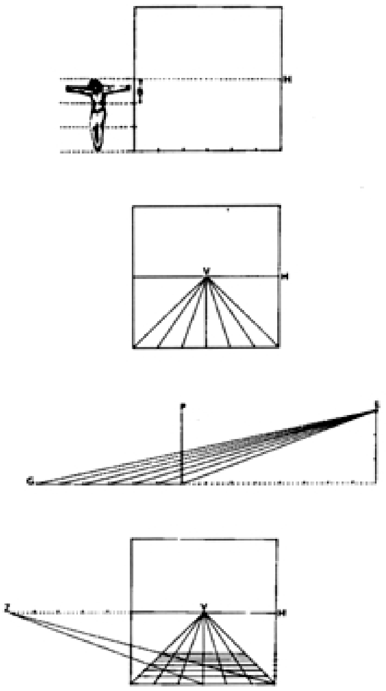


One of Durer's Perspective Machines
Kurth, Plate 338

Project: Make a perspective machine. Use it to make a perspective picture.

The *Costruzione Legittima*

Alberti gives the following method for constructing a picture of a horizontal *grid,* like the paving stones in a plaza, which has come to be called an *Albertian Grid.* In his description he has the orthogonals and transversals both spaced by one *braccio,* a Florentine unit of measure equal to 1/3 the height of a person, about 23 inches. His construction is called the *Costruzione Legittima.*



Alberti's Construction 1.B-one *braccio* module (one third of the height of a man). The base of the picture is divided into *braccia.* The height of the man at the front plane of the picture gives the level of the horizon, H.

2. The *braccio* divisions are joined to the perspective focus, V, to give the orthogonals.

3. In side elevation, lines are drawn from *braccio* divisions behind the picture plane P to the eye at E. The points of intersection on P are noted.

4.The levels of the points of intersection are marked at the side of the picture plane, and locate the horizontal divisions of the tiles. Z is the 'distance' point, though Alberti only mentions using one diagonal to check the construction.

Kemp, p. 23

Project: Draw a pavement before being shown any perspective construction. Then draw a pavement after being shown the method. Compare the two.

Project: Reproduce Alberti's *Costruzione legittima.*

From Euclid to Desargues

Roots

Like most discoveries, perspective theory did not emerge out of a vacuum. The underlying ideas had been accumulating for centuries. While the main application of perspective is in art, it is an optical phenomenon and thus has its principal root not in art but in geometrical optics.

Euclid's *Optica*, C. 300 B.C., was the first text on geometrical optics, in which are defined the terms *visual ray* and *visual cone.*

Vitruvius' *Ten Books on Architecture* which appeared about 25 B.C., was the only book on architecture to survive from antiquity. It profoundly influenced Renaissance architecture and thinking, including that of Alberti, who quoted Vitruvius in his *Della pittura*. Vitruvius wrote,

Perspective is the method of sketching a front with the sides withdrawing into the background, the lines all meeting in the center of a circle. Unfortunately he didn't elaborate on that. Elsehere, Vitruvius' reference to Greek and Roman stage design, implied an understanding of the vanishing point.

Ptolemy's *Optica,* c. 140 A.D., was another early text on geometrical optics, and included theories on refraction. The *centric ray* is defined by Ptolemy as the ray that does not get refracted. The centric ray, we'll see, is important in the theory of perspective. In his *Geographia,* c. 140 A.D., Ptolemy applies the principles of geometric optics to the projection of the spherical surface of the earth onto a flat surface, to produce a map. He is said to have made the first known linear perspective construction for drawing a map of the world. Ptolemy apparently knew about perspective, but applied it only to maps and to stage designs.

Galen's *De usu partium*, c. 175 A.D., contains an early but erroneous description of how the eye creates images. The book was still important, however, as a stepping stone in the development of the theory of perspective.

From Islam, Alhazen's *Perspectiva*, c. 1000 A.D., was an important compendium on optics. It integrated the works of Euclid, Ptolemy, and Galen.

Roger Bacon's *Opus Majus,* c. 1260 A.D., included a section on optics, whose geometric laws, he maintained, reflected God's manner of spreading His grace throughout the universe.

John Pecham's *Perspectiva communis,* c. 1270 A.D., was another treatise on optics that was widely available during the Renaissance. Finally, Blasius of Parma's *Quaestiones perspectivae*, c. 1390 A.D., was a popular adaptation of the works of Bacon and Pecham.

Commerce in Florence

Another element that set the stage for the development of perspective was the rise of banking and commerce in thirteenth century Florence, accompanied with widespread knowledge of arithmetic, especially the figuring of proportions, with geometry to allow a merchant to gauge volumes of wine casks and piles of grain, and of simple surveying for the division of land.

What does bookkeeping have to do with art? It is probable that the tidy mathematical ledgers of the Florentine merchants, made them more receptive to the tidy mathematical organization of space afforded by linear perspective, and the geometric constructions of perspective resonated with the familiar geometric constructions of surveying. While this did not directly contribute to the development of perspective, it did provide a receptive climate.

Mirrors

Slide 12-23: JAN VAN EYCK: ***Arnolfini Newlyweds,*** **or** ***Wedding Portrait,*** **1434**

American Library Color Slides Co. Inc., Cat. #893

Another contributing factor to the development of perspective was the current interest in mirrors. The flat, lead-backed mirror was introduced in the thirteenth century and apparently fascinated artists and writers as much as it did those interested in optics.

Dante made several references to mirrors. Giotto is said to have painted "with the aid of mirrors." Alberti recommends looking at a painting in a mirror to expose its weaknesses. In his *Della pittura* he writes "*A good judge for you to know is the mirror. I do not know why painted things have so much grace in the mirror. It is marvelous how every weakness in a painting is so manifestly deformed in the mirror."*

The mirror shown in the famous painting of the Arnolfini Newlyweds of Jan Van Eyck, was said to have been a standard piece of furniture in the studios of late medieval painters.

This increased use of mirrors by artists generated interest in geometrical optics and provided a way to see a real scene on a flat plane. The convergence of parallel lines to a vanishing point when seen in a real scene is easily ignored because it is so familiar. But when seen on the unfamiliar flat surface of a mirror is less likely to go unnoticed. This may have caused artists to look for the same phenomenon in the real world.

From Euclid to Desargues

Perspective is an example of the geometric operation of projection and section where projection lines from the outline of an object to the eye are sectioned or cut by a picture plane. This has roots in the conic sections, where projection lines from a circle to a point form a cone, which is then sectioned by a plane to give a circle, ellipse, parabola, or hyperbola, depending on the angle of the cutting plane. These ideas were expanded by Gerard

Desargues (1593-1662). Architect, engineer into the branch of mathematics called *projective geometry.*

What Next?

Slide 12-28: PISSARRO *In the Wood,* 1862

Sandak Slides, Inc

Slide 12-30: MONET:*Street in Ste. Adresse,* c. 1869

Sterling & Francine.
Clarke Art Institute, Williamstown MA.

We see from these pictures that perspective was alive and well right up to the end of the ninteenth century.

Slide 12-31: SARGENT:*A Street in Venice,* 1882

Sterling & Francine.
Clarke Art Institute, Williamstown MA.

Slide 12-32: HOMER
***Sleigh Ride,* 1893**

Sterling & Francine. Clarke Art Institute, Williamstown MA.

And then came the twentieth century, with cubists, futurists, constructivistsm Dadists, minimalists, and so on, who said, this is not a window! Stupid! Its a piece of canvas with paint on it. And along came the notion of *integrity of the picture plane*, and pictures got flat - so flat that art critics even argued over the thickness of the paint, and some artists thinned their oils with turpentine to eliminate even that thickness. And and perspective was dead, at least for the avante-guarde, although illustrators and other artists used it as before.

Slide: KASIMIR MALEVICH:*Samovar*

citation

Summary

So we have seen the picture frame turned into a window frame and that linear perspective, so familiar now that we hardly notice it in a picture, had its roots in the developments in geometry, optics, and cartography, between the first and fifteenth century.

Then early in the 1400's the first known perspective picture was made by Brunelleschi, and perspective theory was described in writing by Alberti.

The perspective construction was then used by Donatello, Masaccio, Mantegna, and others, who produced a flood of perspective paintings and sculptures during the Renaissance. Their use of perspective was not only recognized and appreciated by their audiences.

We then saw examples of the use of perspective right up to the twentieth century when we'll later see the reasons for its decline.

Reading

Alberti's *On Painting,* pp. 43-59

Alison Cole, pp. 6-21

Kemp, *The Science of Art*, Chap. 1

Additional Bibliography:

Ivins, Edgerton, Pozzo, Dunning, Vredman de Vries, Rex Cole, Kubovy, Vasari on Brunelleschi, Donatello, Masaccio, Mantegna, Ucello,

Projects

- Reproduce the Peepshow demonstration.
- Xerox a few paintings. Draw orthogonals and locate VP. Speculate as to why the artist placed it there.
- Make a camera obscura.
- Make a perspective machine. Use it to make a perspective picture.
- Draw a pavement before being shown any perspective construction. Then draw a pavement after being shown the method. Compare the two.
- Reproduce Alberti's *Costruzione legittima*.

WHAT SHAPE FRAME?

Slide 13-52: Claude Lorrain, ***A Rest on the Flight into Egypt*** @

We've looked at the plane figures; polygons like the square, rectangle, and hexagon, as well as the circle. We've examined their geometry and shown some of their uses in sacred geometry and in art and architecture. In this unit we'll explore these figures as shapes of frames; how and why a particular shape was used and how it affects the enclosed picture or sculpture.

We'll briefly track the development of paintings from no frame, to open frames, complex frames, and multiple frames, and the altarpiece. We'll show four reasons for having a closed frame, and see how some artists have used axes of symmetry, zones, diagonals, and microthemes.

We'll explore several rectangular formats, the square, double square, overlapping square, and the golden rectangle, and the round format, in painting, sculpture, and medallions, and finish with the elliptical format.

Why Frame a Picture?

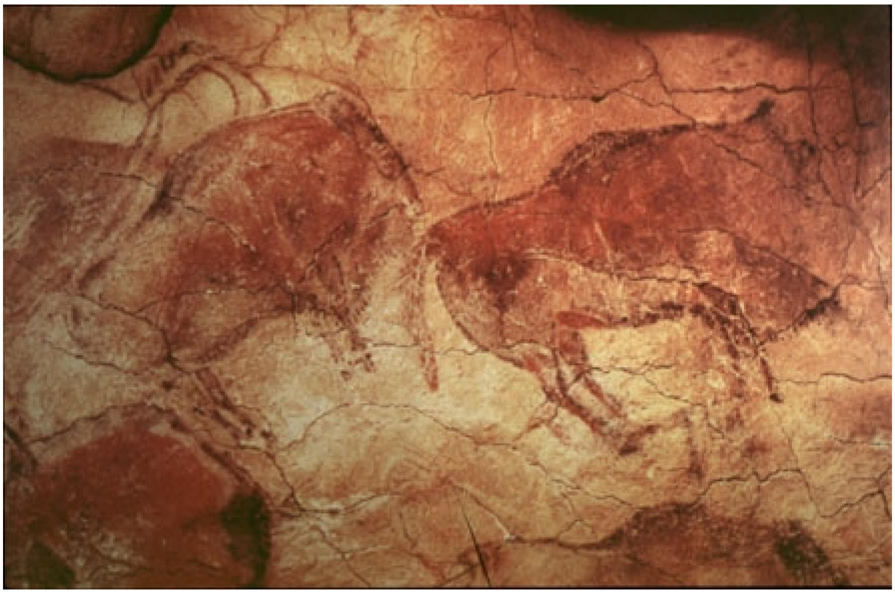
In our last unit on perspective we saw that one function of the frame was as a *window frame* through which we viewed the world. Other purposes for the frame are:

1. *To separate inside from outside*; to state that a picture is a world of its own, and not a part of the surrounding world.

2. *To provide visual control.* This separation from the outside world makes it possible for the painter to control the composition. The meaning of things we see depend partly on their surroundings, so if the surroundings can't be controlled, the meanings cannot be controlled either.

3. *For portability*

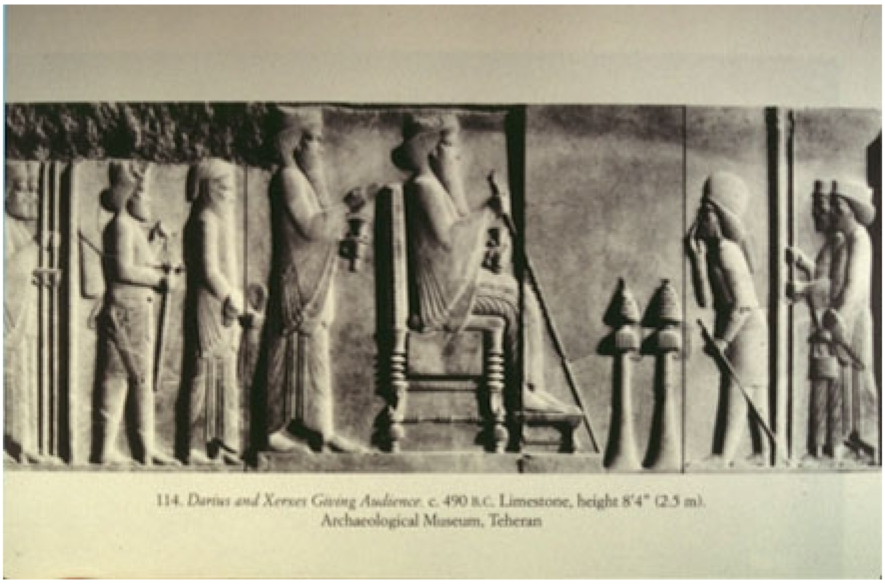
No Frame: Prehistoric Wall Paintings.



Slide 13-1: Cave Painting of a Bison, Altamira, Spain c. 15,000-10,000 B.C.E., Hartt p. 38

Of course, not all pictures were framed. Cave paintings are an example of unframed pictures which generally focus on a single figure or animal, without much regard to the surrounding figures.

Partial Frame: The Frieze



Slide 13-2: ***Darius and Xerxes Giving Audience***, Persian, c. 490 BCE. Janson p. 95

An example of what might be called a *partial frame* is the frieze, often found in the temples. It is closed at top and bottom and open at the ends.

Tapestries and Scrolls

Slide 13-6: ***The Battle of Hastings,* detail of the *Bayeux Tapestry.*** c. 1073-83. Janson p. 325

Other open frames include Oriental scrolls or tapestries, like the famous *Bayeux Tapestry*, depicting the Battle of Hastings.

Closed Frames: Murals and Frescos

Slide 13-8: GIOTTO, Scrovegni Chapel Frescos. Electra p. 11

Murals and frescos might be open, like the Oreszco Frescos, or have clearly defined outlines like in the Scrovegni Chapel.

Time Frames

Slide 13-9: GHIBERTI, Baptistery North Doors, 1403-24, Scenes from the New Testment

A frieze, mural, or fresco might sometimes have a time progression as well as a spatial one, a property that relates them to music. The individual pictures are like the frames in a movie film or pages in a book.

Frames have taken all sorts of elaborate shapes; lancets, Gothic Arches, and quatrefoils as in the Baptistry North doors shown here.

The Altarpiece

Slide 13-11: FRA ANGELICO, *Deposition.* C. 1440.

Perhaps the most elaborate frames of all were the Medieval and Renaissance altarpieces, with their mountain ranges of pointed peaks. They look pretty old to us now, but Jacob Burckhardt writes that the great tradition of modern European easel painting started with the Italian Renaissance altarpiece. The altarpiece was on the cutting edge of Italian painting, using the most advanced methods and materials.

There were no rules for the design of an altarpiece, so it was a place where artists could experiment with new methods and materials. Altarpieces show continual change over the years, both in shape and subject matter. They were regularly replaced by the latest model, the old altarpiece often cut up and pieces scattered in various galleries.

Slide 13-12: DUCCIO, *Maesta Center panel,* 1308-11. Siena. Janson p. 369

Duccio's super-altarpiece, the *Maesta* was a big deal in 1311. According to Hartt, it was carried from his studio to the cathedral in a triumphal procession, the people carrying lighted candles and torches, to the sound of all the bells of the city, and the music of trumpets and bagpipes. But Vasari writing in 1550 says he could not even locate the old altarpiece. But we can see most of it today in the Cathedral Gallery in Siena a few years ago.

There's also a piece of the Maesta in the National Gallery in Washington and another bit in the Frick Collection in New York.

Portability

Most of the pictures we've seen are fixed in place, painted on a wall or carved on a pediment. But during the Renaissance the growing desire of people to buy and own paintings and other art objects loosened the connection between an artwork and its location. According to Arnheim, paintings, religious and mythological subjects, landscapes, genre scenes, could be made in the studio for none in particular and sold and eventually resold. They were part of what we now call the *art market.* For this, pictures had to be *portable,* and we add portability to our list of reasons for framing a picture.

The Switch to Oils and Canvas

Portability may have been one reason to switch from paintings on wood panels to canvas.

Most of these altarpieces were painted with egg tempera on wood panels, standard for altarpieces until at least the 1520's. Canvas had been widely used in Venice for large-scale painting since the 1470's. It was cheaper than wood, and more portable. This made it better for the growing art market of the 1500's.

At about the same time period there was a shift from egg tempera to oils. To make paint, an artist had to grind pigment from minerals or plants into a powder, and add a liquid to make a paste. If egg was used we get *egg tempera*, which was used as early as the 12th Dynasty in Egypt, (about 2000 B.C.E.) and continued in use during through the Renaissance. It was fine, but dried quickly.

Jan Van Eyck, sometimes called the discoverer of oil painting, wanted a slower drying paint to produce the smoother transitions of colors that he wanted. So he substituted oil for egg, and got a slower drying mixture. It also allowed making of glazes.

The *Quadro* or Rectangular Format

Slide 13-17: A Sunday on Le Grande Jatte - 1884. 1884-6, George Seurat (1859-91)

We've seen what frames do. The next question is *what shape to use?* The shape of a frame is often called the *format.* We'll talk about the square format, and the round format. But first, the rectangular format

According to Fisher, as artists turned from painting on wood panel to canvas, the use of the rectangular format increased because it is easier to cut wood into interesting shapes than to construct frames in these shapes and stretch canvas over them.

Once we have decided to use a rectangle, we must decide on (1) orientation and (2) proportions of that rectangle.

Portrait or Landscape

By orientation, we simply mean whether the long side of the rectangle is vertical or horizontal. Of course, a landscape normally calls for a horizontal format; a portrait calls for a vertical format. Computers even use the terms *portrait and landscape* to distinguish between the vertical and horizontal formats.

The orientation may affect how the contents are seen. For example a standing figure painted in a portrait format appears taller and thinner; a standing figure painted in a landscape format is accentuated. It stands out more, because of the contrast with the horizontal frame.

What Shape Rectangle?

So once you decided whether to orient it portrait or landscape, what proportions do you make it?

Within the rectangular format, there's one format for movie films, another format for wide-screen movies, and for video, others for different kinds of photographic film, and still others for paper, like the 8.5 x 11 format.

Golden Rectangle

Slide 10-38: Botticelli, *Birth of Venus*

Why not use the golden rectangle for every painting, as it is often claimed to be the most esthetically pleasing. These claims seem to be based mostly on the work of Gustav Fechner in the 1860's, who presented people with rectangles of white cardboard on a dark table. In this test they tended to prefer proportions approaching the golden ratio (**1.62 to 1**). But, when measuring frames in a museum, Fechner found that the favorite ratios were:

for vertical pictures 5:4 or **1.25 to 1**

for horizontal pictures about 4:3 or **1.33 to 1**

In other words, artists used frames more compact than ϕ, the vertical more so than the horizontal.

Why did artists prefer a squarer shape? My own opinion comes from the very title of Arnheim's book, *The Power of the Center.* As we move from the center things lose importance. Also, as we'll see, a squarer format is consistent with rest, repose, dignity, and timelessness; things that artists often want their paintings to convey.

Overlapping Squares

Slide 13-19: Saint Francis Before the Sultan. Giotto (1267-1336/7), Santa Croce. Fl.

So most rectangular pictures are made with no particular proportions, but there is one that appears often enough to mention; a rectangular painting made up of two overlapping squares.

In this painting by Giotto, the inner rectangle is formed by two overlapping squares with sides equal to the short side of the outer rectangle. Also, the crossings of the diagonals fix the base of the throne and the height of the partition in the background.

Axes and Zones

Slide 13-21: Botticelli's Annunciation c. 1492

A rectangle has two axes of symmetry, vertical and horizontal. An axis of symmetry is often used to divide a painting into distinct regions. Here the vertical splits scene into two regions, the natural region of the Virgin, and the supernatural region of the archangel Gabriel.

The Diagonal

Slide 13-22: TIEPOLO, ***Apollo Pursuing Daphne***, 1755-60. Fisher p. 156.

Another compositional device is to use the main diagonal to make the picture more dynamic. The diagonal of a rectangle is not an axis of symmetry but it also is sometimes used to split the picture into zones, as in the Tiepolo, with the God on one side and the mortals on the other.

The Square Format

Slide 13-26: Donatello: ***Madonna of the Clouds*** c. 1427

The square is a special case of the rectangle, and artists have used some of the same devices, such as using the diagonal to separate a picture into two zones. But unlike the rectangle, the diagonal of a square *is* an axis of symmetry.

But the square format has one property that the rectangular does not; it gives a scene stillness and serenity, a calm and dignity which we'll see again in the round format. This makes it ideal for a subjects such as a Madonna

Raphael: Knight's Dream, or Dream of Scipio

Slide 13-27: The Knight's Dream Raphael, 1504-5, Old Janson, p. 124

We saw a Raphael before, his *School of Athens* of 1510-11. Here we look at two earlier works.

Here again we have a division into zones, but this time by the vertical axis. The vertical tree divides the picture into two zones; virtue and beauty.

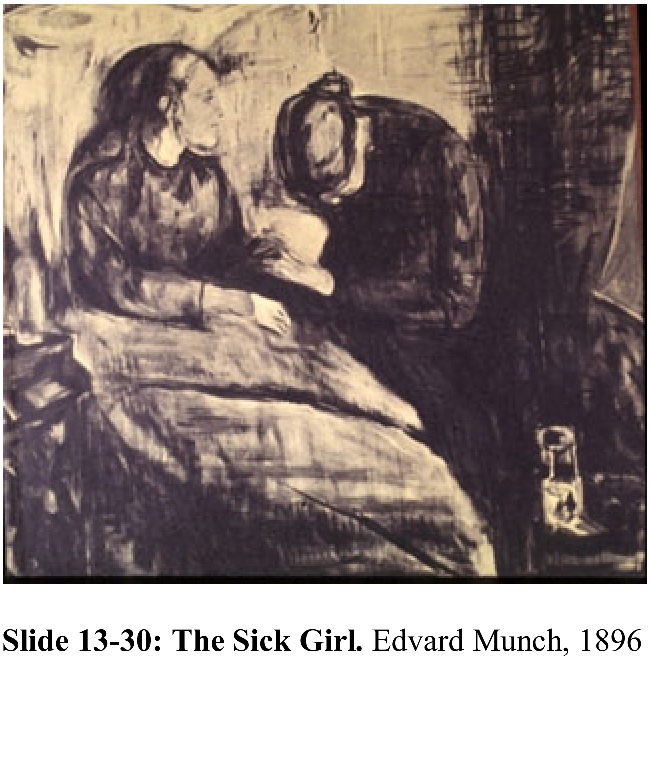
This picture also shows what Arnheim calls a **microtheme**, a miniature version of the main theme acted out near the center of the picture, often with the hands. In *the Knights Dream* the microtheme is the choice between book and flowers, representing the larger choice shown in the painting, the choice between virtue and the pleasures of beauty.

Raphael: Deposition

Slide 13-28: Deposition, Raphael, 1507

Arnheim says that what might be a dynamic scene here is made static by the square format. The microtheme is one support: the gentle raising of Christ's hand by Mary Magdalene, and the young man's strong grip on the cloth. This is an act of raising up, foreshadowing the ascension.

Munch: The Sick Girl



Slide 13-30: The Sick Girl. Edvard Munch, 1896

Here we have a strong diagonal separating the realm of the sick daughter from that of the grieving mother. The microtheme at the very center, where the mother reaches for the daughter's hand but can no longer touch it.

The *Tondo* or Round Format

Slide 37: ***Madonna of the Pomegranate,*** Sandro Botticelli, 1487. Arnheim p. 77.

A round painting is often called a *tondo.* The tondo was also called a *Desco da Parto,* or *Painting on a platter.*

If the center was more important for the square than for the rectangle, it is even more important for the circle where all points on the circle are equidistant from that center. So its not surprising that a microtheme located near the center is more common in the Tondo than in the square.

Another characteristic of the circular form is *detachment.* We have seen that one purpose of a frame is to define a picture as a closed entity, separate from its surroundings; a world of its own.

According to Arnheim, a *round* frame does this best because it strengthens the center, making the picture more self-referential, while a rectangular or square frame does it less well because its sides strengthen the reference to the vertical and horizontal of the outside world.

This detachment makes the tondo a natural choice for a religious picture. Its separation from the secular and vulgar makes it the all-time favorite format for the Madonna.

In the ***Madonna of the Pomegranate*** the Christ child's head is near the center. This formal placement already makes him appear the ruler. The microtheme here is in the upraised hand in blessing, and the handling of the pomegranate, like an orb of power. This clearly give the child a dominant position.

The Roundel

Donatello

Slide 13-41: Virgin and Child with Two Angels. Donatello & assistants. Marble. 1457-8, Cathedral, Siena. Pope-Hennessey p. 84.

As with circular paintings, the circular relief sculpture or **roundel** became popular in the fifteenth century. It was also a favorite shape for religious figures, especially the Madonna.

The Della Robbias

Slide 13-42a: St. John the Evangelist. Luca Della Robbia, (1400?-1482) Enameled terra cotta. c. 1442. Pazzi Chapel. P-H p. 110.

Luca Della Robbia was famous for his works in terra cotta, as was Andrea, Luca's nephew, pupil, and successor.

Slide 13-43: Foundlings. C. 1487. Enameled terra cotta. Andrea della Robbia (1437-1528) Spedale degli Innocenti. PH p. 178.

These are two of 14 *bambini* Andrea made the children's hospital in Florence. Lucca's sons Giovanni and Girolamo were also terra-cotta sculptors.

The Italian Plaquette

Slide 13-45: Mars with Trophies. Moderno. P-H p. 202

Tondi and roundels are symbols of the loosening of the connection between artworks and their locations. As more works of art became portable objects made for the art market, the ultimate example of that was the plaquette.

They were small, single-sided bronze reliefs. They were made in Rome, Florence, and Padua from around 1450 to 1550. Many were rectangular or oblong, but most were round, usually under 6 in. in diameter.

Some themes were religious, but many were drawn from mythology, as this figure of Mars by Moderno, the most prominent and prolific sculptor of plaquettes.

Slide 13-46: Allegory of Virtue. Riccio. PH p. 213

Riccio (1470?-1532) was also active in making plaquettes, like this *Allegory of Virtue.* With many plaquettes the aim was to portray principles of conduct; courage, rectitude, constancy decency, restraint, and virtue.

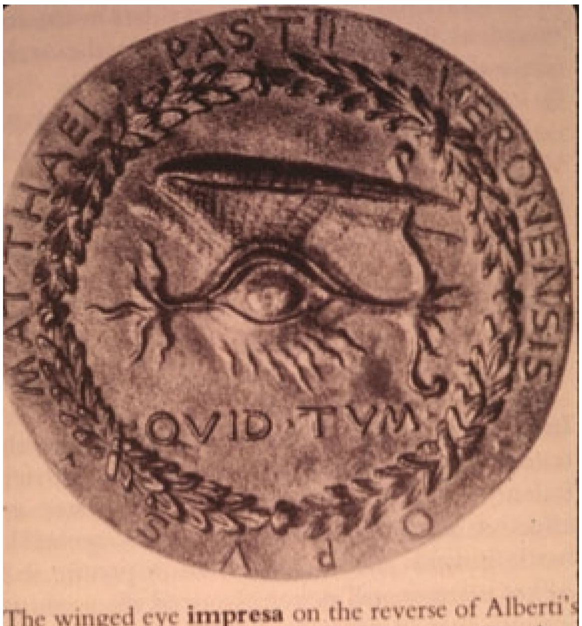
Renaissance Portrait Medals

Slide 13-47: Alberti Medal

Slide 13-48: Medal of Vittorina da Feltre by Pisanello. Levey p. 75.

Here is an example of the Renaissance Portrait medal, objects inspired by antique Roman coins, but purely commemorative. Its greatest practitioner and virtual originator of the Renaissance medal was Pisanello (Antonio Pisano, active c. 1415-55).

The typical design had a profile portrait on obverse, another design on reverse, and an inscription around rim.



The winged eye **impresa** on the reverse of Alberti's medal by Matteo de' Pasti (1446–50) (London, British Museum).

Slide 13-49: Alberti Medal Verso

On the reverse side, an allegorical allusion or an emblem could portray mental characteristics or the essence of the person portrayed on the obverse. For example, the reverse of the medal of Marsilio Ficino, the great Florentine Platonic philosopher, bore only the single word, *Plato.* Medals often bore the family *Impressa,* a kind of trade mark, logo, or insignia, like the *winged eye* on the reverse of the Alberti medal.

Michael Level writes, *"At once a new artifact but with antique echoes, personal, naturalistic and yet allusive, easily portable yet particularly durable, the portrait medal is a perfect symbol of Renaissance endeavor and achievement."*

Here, a few inches of metal held the promise of eternity. It is a perfect example of the Egyptian definition of the word *sculptor, one-who-keeps-alive.*

Ovato Tondo or Elliptical Format

Slide 13-50: Prud'hon, c. 1803

Arnheim says that the Renaissance cherished the circle as the shape of cosmic perfection, the Mannerist and Baroque periods favored the ellipse.

Slide 13-53: A Cameo

The upright ellipse was good for portraits. According to Arnheim, A *... for the portrait, the oval lends welcome assistance in the painter's struggle with the human figure, which carries the head high above its center. The upper focal point of the ellipse offers the head ... a compositional resting place not available in either the tondo or the rectangle."*

In US currency, Presidents are in elliptical frames. Also note ellipses around the numerals on $1, $5, and $20 bills; elliptical frame around Lincoln Memorial on $5 bill.

Summary

We have briefly tracked the development of paintings from

... no frame, as in cave paintings ...

... open frames, as in the frieze ...

... and multiple frames, often showing a time sequence

... and the altarpiece.

We have mentioned four reasons for the closed frame ...

... to separate the world of the picture from the rest of the world

... to control the composition of the picture

... for portability

... as a *window frame* through which we see the world.

We've seen that the emerging art market required portability, which encouraged the use of canvas, which in turn may have led to the use of the rectangular format.

And have explored several rectangular formats; the square and the overlapping square, and have seen that the Golden Rectangle was not a big deal in art. We have seen how some artists have used axes of symmetry, zones, diagonals, and microthemes for compositional purposes.

We then explored the round format, in painting, sculpture, and medallions, and the ellipse.

Reading

Arnheim, Power of the Center, pp. 51-71.

Bouleau Chapter II, *The Frame*. pp. 30-47

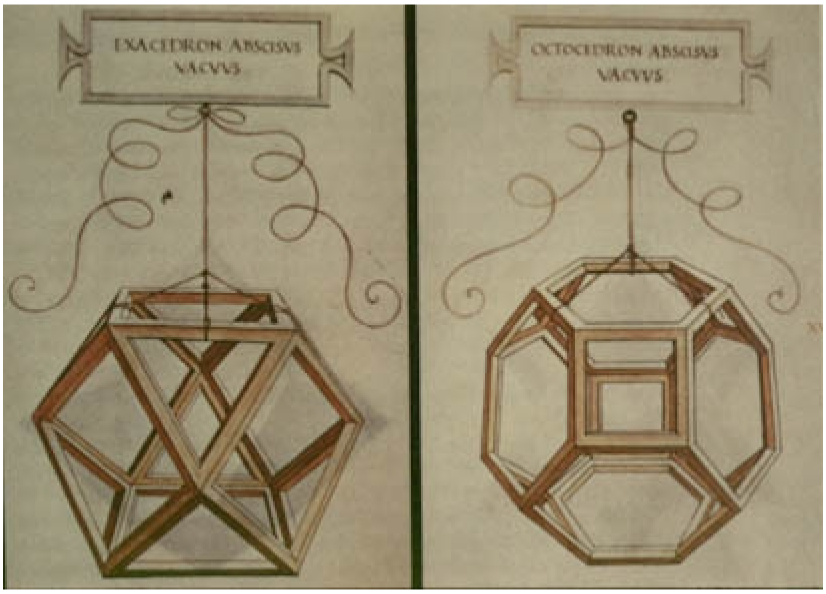
J. Burchardt, *The Altarpiece in Renaissance Italy* (1898), ed. and trans. P. Humfrey, Oxford, 1988, p. 81.

Pope-Hennessey p. 192-222

Levey, *Early Renaissance*, p. 73-77

Review of *The Altarpiece in Renaissance Venice*, Art Bulletin, March 95, p. 139.

POLYHEDRA & PLAGIARISM in the RENAISSANCE



Slide 14-1a: The Regular Solids, one of Leonardo's Illustrations for Pacioli's *Divina proportione*, Reti p. 70.

Piero della Francesca was one of the most original men of the renaissance. Piero had two passions - art and geometry - the very things that this course is dealing with. He carried these on at the same time and, we will try to show here, how he integrated the two.

Luca Pacioli (c.1445-1517) was a renowned mathematician, captivating lecturer, teacher, prolific author, religious mystic, and acknowledged scholar in numerous fields. He was a link between the Early Renaissance of Piero and the High Renaissance of Leonardo.

Leonardo (1452-1519) is considered by many the ultimate renaissance man, skilled in mathematics, philosophy, civil and military architecture, engineering, painting, sculpture, science, music, invention, and the design of weapons. We'll consider Leonardo in our next unit.

These three Renaissance men were linked as student to teacher; Piero taught mathematics to Luca, and Luca taught Leonardo. They are also linked by the Platonic solids.

Piero della Francesca

Slide 14-1: Statue of Piero in Sansopolcro, Piero's birthplace.

Piero della Francesca (1420-1492) had two passions - art and geometry. He sought that link between an organic and geometric basis of beauty, what Kenneth Clark has called *the Philosopher's stone of aesthetics.*

De Prospectiva pingendi

Piero was one of the greatest practitioners of linear perspective, and even wrote a book on perspective, *De Prospectiva pingendi* (c.1474). This was his first of three books, the others were *Trattato del abaco*, and *De quinque corporibus regularibus*, both of which we'll mention later.

Piero wrote his book on perspective thirty-nine years after Alberti's *Treatise on Painting* of 1435. It is considered as an extension of Alberti's, but is more explicit. Piero was evidently familiar with Euclid's *Optics*, as well as the *Elements*, whose principles he refers to often. Theory is fine, but did Piero practice what he preached? According to Martin Kemp,

"The evidence of his paintings suggests that he did exercise an . . . extraordinary degree of meticulous, time-consuming, geometrical care over the perspectival projection of architectural forms. . ."

The *Flagellation of Christ*

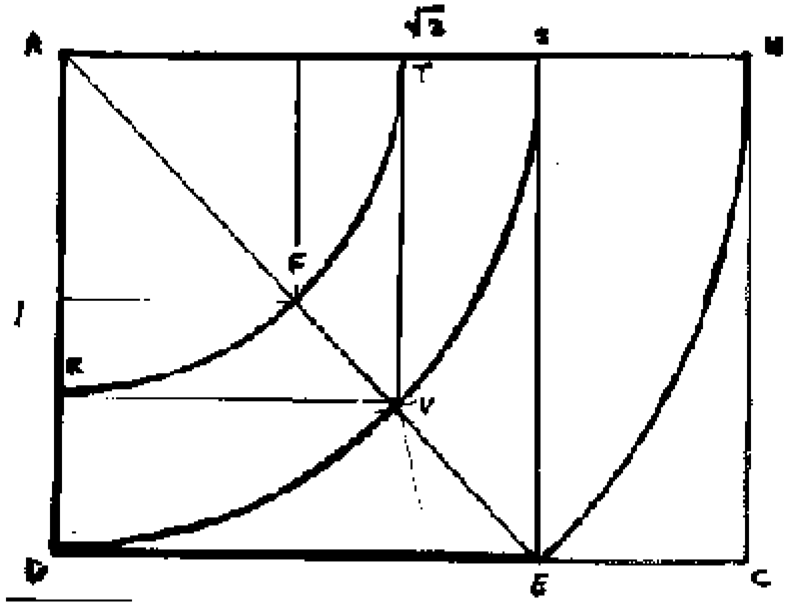
Slide 14-2: *Flagellation of Christ*, c. 1460

Let's pick up the thread with Piero's *Flagellation*, the slide with which we closed the unit on perspective. Talking about this painting, done fourteen years before he wrote his book on perspective, Kemp says,

"No picture could exude a more pronounced air of geometric control and no painting was ever more scrupulously planned."

This painting has been analyzed to death, and I even have a computer analysis locating the vanishing point to the nearest millimeter, in a painting that is 81,500 mm by 59,000 mm.

If you extend the orthogonals to locate the vanishing point, it does not seem to fall on any point that is significant to the story. But Kemp shows that point does have *geometric* significance.



Slide: Geometry of the Flagellation

He gets his clue from the dimensions of the painting; 58.4 cm (one *braccio*) by 81.5 cm., giving the ratio of the sides = 1.40 ~ $2^{1/2}$. If we swing arc EB from A, we get a square ASED. Thus the width of the painting equals the diagonal of the square, verifying that the frame is a root-two rectangle.

The diagonal AE of that square passes through the V, vanishing point of perspective.

Further, in square ATVK we see that arc KT from A cuts the diagonal at Christ's head, F, halfway up the painting. Thus Christ's head is at the center of the original square ASED.

The Tile patterns:

Christ stands on a simple green circle, inscribed in a square. A case of squaring the circle?

Kemp points out that the tile pattern *inside the praetorium* is complex, and based on the diagonal of a square, an irrational number such as found in the proportions of the frame. The tile pattern *outside the praetorium* is simple squares, in an Albertian grid, and there is no apparent correlation between them. This seems to create *two separate zones*: the space inside, occupied by Christ, and the more mundane world outside. Deliberate? Kemp says that for someone as subtle as Piero, this is not impossible.

Further, the tiles *inside* the praetorium form an octagonal pattern. Recall from our unit on number symbolism that the octagon suggests resurrection and baptism, because Christ rose from the tomb eight days after entering Jerusalem.

---

de Abaco and Mathematical Education in the Renaissance

That was pretty subtle geometry in the *Flagellation*. Could a Renaissance viewer really appreciate all that? How much mathematics did the average person know?

In Renaissance Florence, according to Baxandall, a boy was educated in two stages. For about four years, from age six or seven, he learned reading, writing, and business correspondence. Then for another four years most would go to a secondary school, the *abbaco* (which means *arithmetic*), where they learned mostly mathematics, especially commercial mathematics adapted to the merchant.

One of the books used in these schools was Piero's second book, *de abaco*.

Proportion and the *Rule of Three*:

One important subject was how to solve proportions, crucial to a merchant who had to deal with problems of pasturage, brokerage, discount, tare allowance, adulteration of commodities, barter, and currency exchange. Not only did every city have its own currency, but its own weights and measures!

The universal mathematical tool of literate commercial people in the Renaissance was the *Rule of Three*, also called the *Golden Rule* and the *Merchant's Key*. In his *Del abaco*, Piero explains how to use the rule of three to solve a proportion:

"multiply the thing one wants to know about

by the thing that is dissimilar to it,

and divide by the remaining thing.

The result is dissimilar to the thing we want to know about."

Example: If seven bracci (1/3 person's height or about 23") of cloth are worth nine lire; how much will five bracci of cloth be worth?

Solution:

The thing we want to know about is:	5 bracci of cloth.
The thing dissimilar to it is:	9 lire
The remaining thing is:	7 bracci of cloth.

So:

(5 bracci) x (9 lire) / (7 bracci) = 6 3/7 lire

The units are lire, because lire are dissimilar to bracci, the units in the thing we wanted to know about.

Skills in Proportion Affected the Way People Saw Pictures

So what does ability to solve proportions have to do with art? Baxandall claims that the skills used to solve exchange problems were the same used for the making or seeing of pictures.

He makes the following points:

1. Renaissance education placed exceptional value on a few math skills, like proportion. People didn't know more math than we do, but they knew it absolutely; it was a relatively larger part of their intellectual equipment.

2. The math skills used by merchants were the same as those used by the painter.

3. The status of these skills in his society was an encouragement to the painter to assert them in his pictures.

4. Because merchants were practiced in manipulating ratios, they were sensitive to pictures carrying the marks of similar processes. It was a small step from the proportions of a currency exchange to the proportion of a physical body, such as the cup and the fish in the slide, to the proportions of a man's head, as done by Leonardo in this sketch from his notebooks, or by Piero in his *Flagellation*.

Familiarity with Gauging Affected the Way People Saw Pictures

Standard sized containers date only from the nineteenth century, so quick calculation of volumes of barrels, sacks, piles of grain, etc., called *gauging,* was a condition of business.

In the 15th century German merchants used complex prepared rulers. An Italian used geometry and π, and Piero's *De abaco* gives methods and exercises for these computations.

As with proportion, according to Baxandall, skill at gauging affected art. The ability of a merchant to break down complex shapes into simpler geometrical units is similar to the way a painter or sculptor analyzes shapes.

Painters and merchants had received the same mathematical education; this was the geometry they knew and used. So the literate public had the same geometrical skills with which to view paintings,

and the painters knew this. We'll see a good example in Piero's *Vision of Constantine.*

Six Paintings by Piero

In this section we'll look at six paintings by Piero and, for each, try to point out anything of geometric interest.

Nativity

Slide 14-2a: The Nativity, Piero della Francesca, 1472-4

Square Format Arrests Motion: Remember from our first session how the square format tended to make a scene static and solemn? Arnheim notes that here the square format turns story of the Adoration into a solemn group portrait.

Axes of Symmetry: We have noted earlier, in Raphael's *Knights Dream* and Munch's *Sick Girl*, for example, that the axes of symmetry of square format often used to divide zones. Here, the central vertical divides scene into two groups; angelic musicians on left, Holy Family and shepherds on right. The two groups do not interact.

Resurrection of Christ

Slide 14-3: The Resurrection of Christ, Piero della Francesca, 1460-63 (Cat. 19806)

We see that Piero was fond of the *square format*. Here's another example, for this painting is essentially square, being about 10% higher than it is wide.

Stillness: The square format, as we've seen before, gives a mood of overall stillness. This static quality is enhanced by locating Christ exactly on center. Arnheim says that Christ's frontal symmetry makes him look like a statue. Christ is static but the guards appear restless. Their tilted heads are a perfect foil for the riveting head of Christ.

Vertical Zones: The central *vertical* divides the scene with Winter on left, summer on right. The rebirth of nature is here probably a symbol for rebirth of Christ.

Horizontal Zones: Similarly, the painting is divided into three *horizontal* bands. Christ occupies the middle band, his head and shoulders reaching into the upper band of sky. His figure unites earth and the heaven. The guards are in the zone below the line marked by Christ's foot.

Baptism of Christ

Slide 14-4: ***Baptism of Christ*****,** priory of San Giovanni, Sansopolcro. c. 1450 (Cat. # 2690)

Arnheim points out the following:

* A *microtheme*, baptism bowl near center of picture.

* Severely foreshortened dove at center of circle and top of square. lends a mystical touch.

* Frozen, static quality found in square pictures, or in round pictures.

* The tree trunk is at 1/3 the width, the dove is at 1/3 the height.

* Piero's home town of Sansopolcro can be seen in the distance, between Christ and the tree.

* John's arm and Christ's loincloth continue the line of the circle.

* Square and circle overlap, like the circles in a vesica, with Christ in area common to both. If the circle is equated with heaven, and the square with earth, here Christ is shown as mediator between the two. Here Piero has combined squaring the circle and the Vesica in one painting

* The recurring motif of the three graces.

* Figures are simplified, cubist, geometric. Piero has been called "the first cubist."

The Introducer

Slide 14-5: *Baptism.* Closeup of the angels

Recall that Alberti, in his *Della pittura,* gives some suggestions for history paintings, or *istoria*.

". . . I like there to be someone in the 'istoria' who tells the spectators what is going on, and either beckons them with his hand to look, or with ferocious expressions and forbidding glance challenges them not to come near, as if he wished their business to be secret, or points to some danger or remarkable thing in the picture, or by his gestures invites you to laugh or weep with them."

In the *Baptism*, this role is filled by the three angels, one of whom makes eye contact with the viewer.

Madonna del Parto

Slide 14-6: C. 1496. Fisher p. 47

Piero's fondness for the square format and for symmetry is again shown here. Note the two angels are practically mirror images.

Vision of Constantine

Slide 14-7: ***Vision of the Emperor Constantine***, from Legend of the True Cross, c. 1450. S. Francesco, Arrezo (Cat. 2449)

According to Baxandall, an obvious way for a painter to invoke the gauger's response was to use the repertory of stock objects from gauging exercises - cisterns, columns, towers, paved floors, and so on.

Almost every math handbook had an exercise to compute the surface area of a conical tent like the one in this painting, to find the amount of cloth needed to make it. A painter who left traces of such analysis was leaving cues that his public was well equipped to pick up. Also, the geometric figure functions as an Albertian *mediator* between artist and merchant, a similar function performed by the angel in the *Baptism*, and by the seated man in this painting.

Also, the tent pole and the horizontal band on the tent form a *Latin cross*, another familiar shape to act as mediator.

Also noteworthy is:

the highly foreshortened angel, who enters the scene from the viewer's space.
Light bursts from him bathing the scene.
Night scenes were very rare in painting at this time.

Piero's last known painting

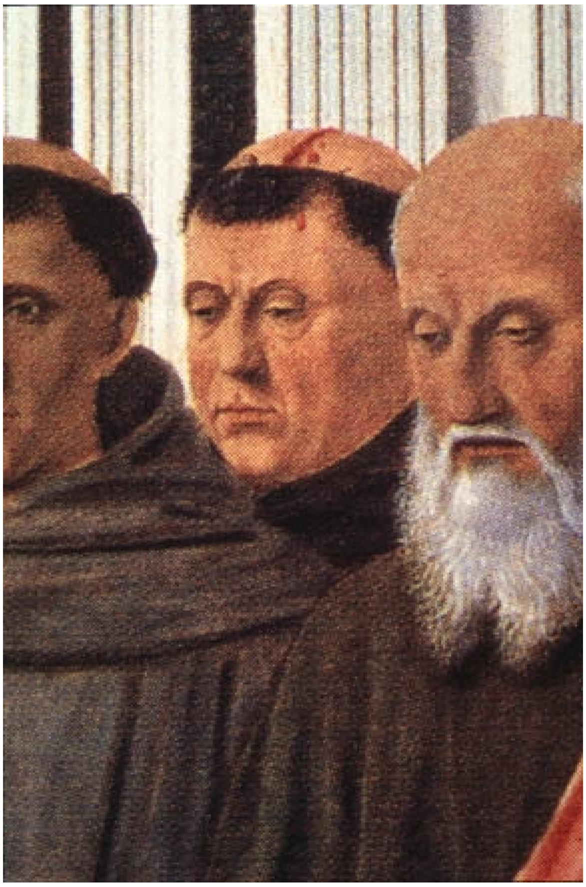
Slide 14-8: ***Virgin and Child with Saints, Angels, and Federigo da Montefeltro***, 1472-4, 248 x 170 cm. Beirati p. 86 (Cat. # 6460)

This painting was commissioned by Federigo, Duke of Urbino, for a new chapel after the death of his young (26 years) wife Battista Sforza. She would ordinarily be kneeling opposite Federigo, and in front of her namesake John the Baptist, but is absent.

A mysterious **ostrich egg** hovers above, the way the dove hovered over Christ in the *Baptism*. Such an egg often hangs over alters dedicated to the Virgin. It was believed that the ostrich let her egg hatch in the sunlight without intervention, and thus became a symbol of virgin birth. Also, the ostrich is here an absent mother, a symbol of the deceased Battista.

The **vanishing point** is on the Virgin's head. Geometrically, the ratio of width to height is 2:3, a **musical ratio.** This same ratio appears in the vault, which has 6 coffers in one direction and 9 in the other. The ratio of the diameter of the circle of the vault to a circle inscribed in the lower part of the picture is also 2:3.

De Quinque Corporibus Regolaribus



Slide 14-9: ***Madonna and Child with Saints and Angels*, detail showing Pacioli.**

Twenty years before his death in 1492, Piero is said to have given up his highly successful career in painting and devoted himself entirely to mathematics. This may have been due to failing sight. Vasari says that Piero went blind at about age 60, (c.1480).

It was during this period that he wrote his three math books, the two already mentioned, and his last, on the five regular solids, *De quinque corporibus regularibus*, described by Plato in his *Timaeus*.

Piero's book was never published but was placed in the library of Federigo da Montefeltro, seen kneeling in this painting. Unfortunately the work was plagiarized. By whom? By another man in this same painting, Piero's student, **Luca Pacioli!**

Luca Pacioli

Slide 14-10 Portrait attributed to Jacapo de' Barberi, closeup. Lawlor cover

Luca Bartolomes Pacioli (c.1445-1517) was a renowned mathematician, captivating lecturer, teacher, prolific author, religious mystic, and acknowledged scholar in numerous fields. He was a link between the Early Renaissance of Piero and the High Renaissance of Leonardo.

Luca's Connections to Piero

Luca and Piero were close. Both were born in Sansopolcro, and Luca was tutored in mathematics by Piero. Luca even posed for Piero in the mid 1470's for *Madonna and Child with Saints and Angels*, in which Pacioli portrays St. Peter Martyr, with cut head symbolizing his assassination.

Piero and Luca sometimes walked across the Apennines to the library of Duke Federigo of Urbino, which was then one of the finest in Europe. Raphael may have been thinking about that library, with portraits of the writers above their works, when he painted the *School of Athens*.

Luca's Connections to Alberti

Piero introduced Luca to Alberti, who arranged for Pacioli's first teaching job, tutor for the three sons of a wealthy Venetian. Alberti, the champion of the Italian vernacular, urged that Pacioli write about mathematics in that language. Alberti had belief in the cosmic significance God-given validity of musical ratios, and this belief was carried on by Pacioli

The *Summa*

In 1472, shortly after Alberti's death, Pacioli took vows, and is usually shown in paintings in a Franciscan habit. Luca lectured in math in Perugia (1475). He then he took to the road (c.1475-1497) and became traveling teacher of mathematics, giving sermons, writing.

He published the *Summa de arithmetica, geometrica, proportioni et proportionalita* (1494) a summary of arithmetic, geometry, and algebra, the sort of book a weak student now uses in a remedial math course. It was one of the first books to be printed in Venice by the new Gutenberg method. It contains the first mention of double-entry book-keeping, for which Luca is now known as the *"Father of Accounting."*

Luca's Connections with Leonardo

In 1497 Leonardo da Vinci, impressed by the *Summa*, apparently encouraged Sforza to bring Pacioli to Milan to tutor him in mathematics, geometry, and proportion. They stayed together for 10 years, in Milan and Florence. Pacioli is mentioned several times in Leonardo's notebooks of this period., and Leonardo got his knowledge of perspective from Piero through Pacioli.

Luca went to Pisa in 1500 , lectured on Euclid, and in 1509 produced a Latin edition of Euclid, based on Euclid's Thirteenth book of *Elements*. In his portrait attributed to Jacopo de' Barbieri, Pacioli's hand rests on the Thirteenth book of Euclid.

The *Divina Proportione*

Luca published the *Divina proportione* in 1509. He wrote it in the Italian vernacular, as urged by Alberti. In it he refers to the golden ratio *"dal ciel mandata"* - heaven-sent. Why does he call the golden ratio *divine*?

1. Like God, it is unique.

2. As the Trinity is one substance in three persons, the divine proportion is a single proportion in three terms.

3. As God cannot be described in words, the divine proportion cannot be expressed by a rational number. It is occult and secret (*secretissima scientia)*.

4. Like God it is always similar to itself.

So after centuries of oral transmission, the golden ratio is finally pinned down in writing.

And what happened? Everyone seemed to lose interest in it.

According to Bouleau, *"Perhaps the turning of a bright light upon old secrets is the annunciation of their death. In fact, it marked the beginning of the decline. The divine proportion had lost its magical attraction ... "*

Lettering

The second main topic in his *De divina Proportione* was classical Roman lettering. It was addressed to stone cutters and builders, and probably influenced later similar works by Dürer and Geofroy Tory.

Polyhedra & Plagiarism

Slide 14-11 Platonic Solids from Pacioli Show

Pacioli devoted the entire second part of *Divina Proportione* to the Platonic solids, relating the Platonic solids to the golden ratio like this:

"As God brought into being the celestial virtue, the fifth essence, and through it created the four solids . . . earth, air, water, and fire ... so our sacred proportion gave shape to heaven itself,

in assigning to it the dodecahedron . . . the solid of twelve pentagons, which cannot be constructed without our sacred proportion. As the aged Plato described in his Timaeus."

Here Luca equates God with Plato's divinity, and wraps up the Platonic solids, the golden ratio, the creation of the universe, and God, into one neat package.

Luca credits Leonardo with the illustrations in *De divina proportione.* He writes, "*the most excellent painter in persective, architect, musician, and man de tutte vertu doctato, Leonardo da Vinci, who deduced and elaborated a series of diagrams of regular solids ...* "

This section of the book has little relation to the earlier part and was apparantly stolen from Piero and tacked on, without credit. Vasari (1550) had some harsh words for the good friar.

"The man who should have tried his best to increase Piero's glory and reputation (since he learned everything he knew from him), instead wickedly and maliciously sought to remove his teacher Piero's name and to usurp for himself the honour due to Piero alone by publishing under his own name - that is, Fra Luca Pacioli, all the efforts of that good old man ... "

Reading

De prospectiva pingendi

De abaco and Mathematical education in the Renaissance

Gauging

De quinque corporibus regularibus

Vasari's chapter on Piero della Francesca

Pedoe, pp. 82-101

Panofsky pp. 129-138

Kemp pp. 27-35

Cole, pp. 18, 19, 24, 25, 28

Reti, 56-85, 216-239

Baxandall, Part II, Sections 9-11 (pp. 86-108) Baxandall, 109-152

Tinius and Weis paper

LEONARDO

"Non mi legga chi non e matematico."

(Let no one read me who is not a mathematician.)

Slide 15-1: Self Portrait

The Ultimate Renaissance Man

Slides 15-2 Leonardo's Birthplace

Leonardo (1452-1519) is considered by many the ultimate renaissance man, skilled in mathematics, philosophy, civil and military architecture, engineering, painting, sculpture, science, music, invention, and the design of weapons.

Slide 15-6: Lorenzo de' Medici. ***The Magnificent***. Terra cotta bust, Verrocchio.

Leonardo got his start in Florence, but he left that city in 1482, at the age of 30. Why did he want to leave Florence, and why did Lorenzo let him go and make no attempt to get him back once he was famous?

Lorenzo encouraged exportation of Florentine artists in 1480's. It provided prestige and opportunities for exchanges, but this exodus caused Florence to lose the artistic lead it had held for 200 years.

Other reasons for leaving were severe competition, war, plague, taxation. Further, Leonardo had contempt for the doctrines of Savonarola. Finally, Leonardo had no use for Lorenzo and his neoPlatonists.

The Plato Academy

Slide 15-7: Bust of Ficino, by Andrea Ferrucci da Fiesole, in Florence Duomo. Encyc Ren p. 134

Most humanists of the Renaissance had a reverence for Plato, and Cosimo de Medici (Lorenzo's grandfather) resolved to make Florence the center of neoPlatonic learning.

In 1463 Cosimo commissioned Marsilio Ficino to translate Plato's *Dialogues* into Latin, and had a villa built for him at Careggi in which to work. There Ficino burned an eternal lamp in front of a statue of Plato. He could recite an entire Plato dialog but, according to his nieces, couldn't remember where he put his slippers. Ficino was the first of the so-called *Plato Four*, the main members of the Medici Platonic Academy, which attracted scholars form all over Italy.

Another was *Cristoforo Landino*, well-known commentator on Virgil, Horace, and Dante, who looked upon Lorenzo the magnificent as Plato's "Ideal ruler."

Slide 15-8: Poliziano, with Lorenzo's son Piero, From Ghirlandaio's fresco cycle in S. Trinita. Encyc. of Ren. p. 260

A third member was *Angelo Poliziano*, ugly but brilliant, publishing in Latin by age 10 and translated Homer by 16. And the fourth was *Pico della Mirandola*, young and attractive, read and wrote in 22 languages, aspired to hold in his mind the totality of human learning. He wanted to reconcile all religions, and attempted to reconcile the creation stories in the *Timaeus* with *Genesis*.

These four, the most brilliant minds in Italy, would meet in Lorenzo's studiolo, often inviting Michelangelo.

Leonardo vs. the NeoPlatonists

Legend has it that Leonardo was Raphael's model for Plato in the *School of Athens*, and he would probably been appalled if he knew of this, for the scientific and artistic Leonardo apparently had little sympathy for the lofty poetic Neoplatonism of the Medici court.

Rohatyn writes, *Platonic aesthetics he had no time for; he was too busy creating perfection to sit back and idly contemplate it.*

And the man often called the greatest genius of all time felt ***inadequate!***

Leonardo, says Italo Calvino, had a difficult relationship with the written word. He was, in his own estimation, an *omo sanza lettere,* an unlettered and uneducated man. His knowledge, continues Calvino, was without equal in the world, but his ignorance of Latin and grammar prevented him from communicating in writing with the learned men of his time. Learned men looked down on him, so he in turn shunned their ideas about the route to wisdom, preferring observation, experience, and experiment to contemplation. In his notebooks he wrote;

"I am fully conscious that, not being a literary man, certain presumptuous persons will think that they may reasonable blame me; alleging that I am not a man of letters. Foolish folks! do they not know that I might retort ... that they, who deck themselves out in the labors of others will not allow me my own ... they do not know that my subjects are to be dealt with by experience

rather than words, and experience has been the mistress of those who wrote well."

The phrase ... *they who deck themselves out in the labors of others* must certainly be a jab at the neoPlatonists, decking themselves out in the labors of Plato.

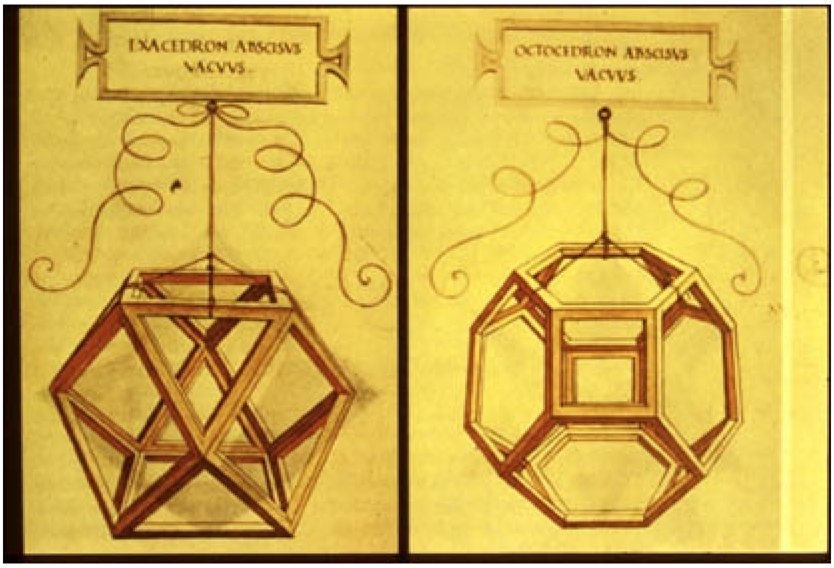
Leonardo's Mathematics

Non mi legga chi non e matematico. "Let no one read me who is not a mathematician."

Recall Plato's inscription over his academy door, *Let no one enter who is lacking in geometry?* This phrase taken from Leonardo's notebooks gives some idea of the importance he placed on mathematics. He also wrote, *"There is no certainty in sciences where one of the mathematical sciences cannot be applied ... .*

The Platonic Solids

Slide 15-11: Leonardo's Illustrations, Reti p. 70



Leonardo studied Pacioli's *Summa*, from which he copied tables of proportions and multiplication tables. Luca, 7 years older than Leonardo, arrived in Milan in 1496, probably at Leonardo's suggestion, and by 1497 they were collaborating on Pacioli's *Divina Proportione*, Published in 1509.

Pacioli devoted the entire second part of *Divina Proportione* to the Platonic solids. *relating the Platonic solids to the golden ratio like this:*

"As God brought into being the celestial virtue, the fifth essence, and through it created the four solids ... earth, air, water, and fire ... so our sacred proportion gave shape to heaven itself, in assigning to it the dodecahedron ... the solid of twelve pentagons, which cannot be constructed without our sacred proportion. As the aged Plato described in his Timaeus."

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Luca credits Leonardo with the illustrations in *De divina proportione.* He wrote,

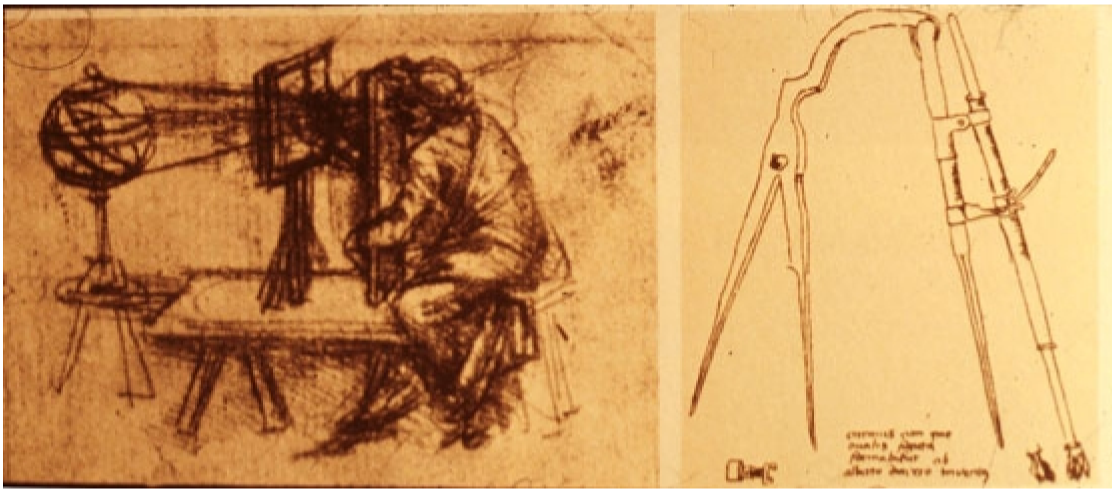
"the most excellent painter in perspective, architect, musician, and man de tutte vertu doctato, Leonardo da Vinci, who deduced and elaborated a series of diagrams of regular solids ... ."

Troubled by his lack of education, Leonardo had an ongoing program of self-study in language and mathematics, studying Pacioli's *Summa,* and Euclid as well.

In his early forties this obsession with mathematics overtook him, and his notebooks began to fill up with geometrical sketches and diagrams. One page shows his studies of the Rule of Three for a problem about weights in a balance. Elsewhere he gave a proof of Pythagorean theorem and gave a rusty-compass construction of 15°. He found the center of mass of tetrahedron and attempted to duplicate the cube.

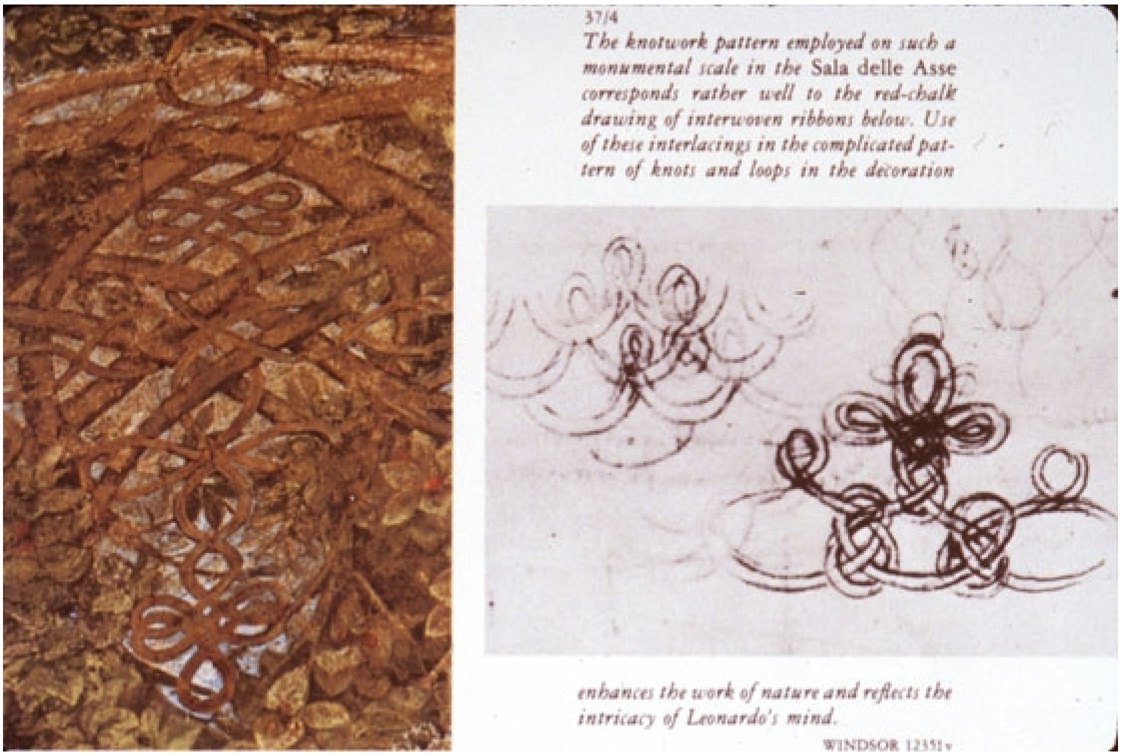
To make the rosette patterns and for other geometric studies, Leonardo used and even invented a wide variety of compasses. He designed a proportional compass that would form a figure similar to, and in a given proportion to another figure. Leonardo gives an interesting way to draw an ellipse, and gives drawings of an ellipsograph

Slide 15-12: Ellipsograph, Reti p. 270



Knots and Meanders

Slide 15-13: Sala delle Asse c.1495-8 text pics Reti p. 37.



Leonardo's knot designs anticipate the modern interest in knots, in the branch of mathematics called Topology. In his famous arboreal decoration for the ceiling of the Sala delle Asse in Sforza's castle, Leonardo made the branches of the trees form an elaborate system of interlacing, like those in his *Academia* engravings. To these he has added a single, continuous rope that intertwines throughout the entire vault.

Leonardo's knot patterns are similar to the Celtic designs such as in the Book of Kells, but more probably were inspired by Arabesques from the middle east.

Vitruvian Man

Slide 15-19: Reti p. 79

According to Augusto Marinoni, *'The problem in geometry that engrossed Leonardo interminably was the squaring of the circle. From 1504 on, he devoted hundreds of pages in his notebooks to this question of quadrature ... that so fascinated his mentor Pacioli ... While his investigations produced no appreciable gain for mathematics, it did create a multiplicity of complex and pleasing designs."*

Slide 15-20: Vitruvian Man Wasserman p. 43

Vitruvius wrote,

"...in the human body the central point is naturally the navel. For if a man be placed flat on his back, with his hands and feet extended, and a pair of compasses centered at his navel, the fingers and toes of his two hands and feet will touch the circumference of a circle described therefrom. And just as the human body yields a circular outline, so too a square figure may be found from it. For if we measure the distance from the soles of the feet to the top of the head, and them apply that measure to the outstretched arms, the breadth will be found to be the same as the height ..."

Clark writes, *"It is impossible to exaggerate what this simple-looking proposition meant to the men of the Renaissance. To them it was far mor than a convenient rule: it was the foundation of a whole philosophy. Taken together with the musical scale of Pythagoras, it seemed to offer exactly that link between sensation and order, between an organic and a geometric basis of beauty, which was the philosopher's stone of aesthetics."*

Chaos & Fractals

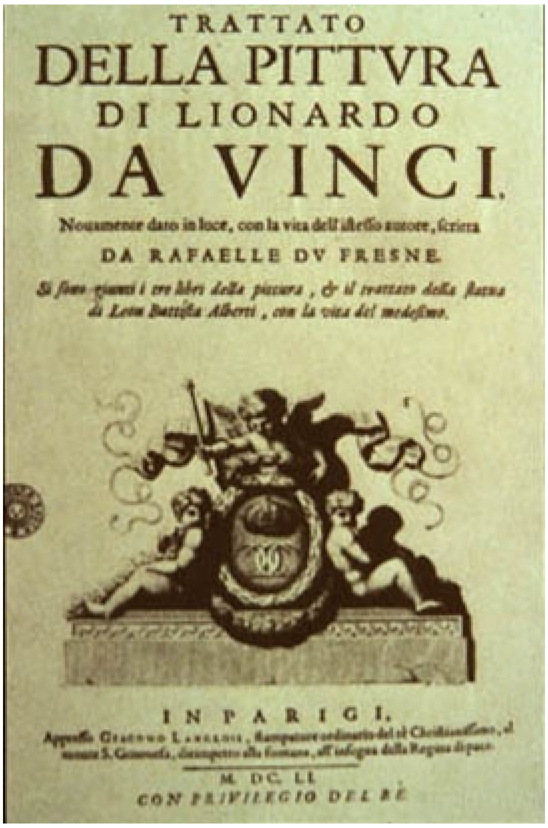
Slide 15-30: Deluge, c. 1514. Mannering p. 68

Finally, Leonardo appeared to anticipate the current interest in chaos and fractals with his drawings of turbulence and of the deluge, which he described in great detail in his notebooks. And listen to him talking about the stains on a wall. *"I cannot forbear to mention ... a new device for study ... which may seem trivial and almost ludicrous ... [but] is extremely useful in arousing the mind ... Look at a wall spotted with stains, or with a mixture of stones ... you may discover a resemblance to landscapes ... battles with figures in action ... strange faces and costumes ... and an endless variety of objects ... According to Augusto Marinoni, confusedly, like the sound of bells in whose jangle you may find any name or word you choose to imagine.*

It almost sound like a description of a fractal pattern.

Leonardo's *Treatise on Painting*

Slide 15-24: The *Trattato*



Leonardo da Vinci was of the few great artists to leave a large quantity of writings; large and small notebooks, pocket books, and separate sheets. They were written in Leonardo's famous mirror-image script, with his left hand. All were left to Francesco Melzi, Leonardo's pupil, friend, and heir, who began the frustrating job of editing the jumble of notes with the aim of publishing them.

Melzi's first and only, project (c. 1550) was to compile a treatise on painting, the *Trattato,* which he never finished, and after Melzi died in 1570 Leonardo's original manuscripts were soon dispersed. Some given away, some stolen, some lost, some sold. Some were cut up for their drawings. Martin Kemp estimates that about three quarters of Leonardo's manuscripts are lost.

Melzi's unfinished manuscript for the *Trattato* found its way to the Urbino Library of Federigo da Montefeltro in 1472, the library we already mentioned that Piero and Luca used to walk to. After Federigo's death the contents of his library wound up in the Vatican Library under the name *Codex Urbinas Latinus 1270*, or simply the *Codex Urbinas* where it lay forgotten until 1817, when Guglielmo Manzi had it published.

The Need for the Book

In the Middle Ages, seven branches of learning were recognized as liberal arts. For a Bachelor of Arts: grammar, logic, and rhetoric: the *trivium.* For a Master of Arts: geometry, arithmetic, music, and astronomy: the *quadrivium*. But the visual arts had been classed among the crafts and mechanical arts because they were "handiwork," and lacking a theoretical basis.

Renaissance artists wanted to break this distinction and towards the end of the 15th century the *botteghe* or workshops of Piero della Francesca, Mantegna, Bramante, Bellini, Verrochio, Pollaiuolo, and so forth, had become small centers of scientific study, where mathematics, anatomy and perspective were learned in an attempt to raise the status of their profession.

There was a lot of empirical knowledge available, but there was no systematic framework in which to organize it. The scientific texts of antiquity and of the Middle Ages were not adequate.

Artists no longer wanted the studio recipes taken from ancient texts. Thus there was a real need for Leonardo's treatise on painting.

The *Trattato* may be subdivided into the following sections:

***The Human Body**:* proportions, anatomy, motion, posture, expression, decorum, and drapery.

***The Depiction of Nature**:* light, distances, atmosphere, smoke, water, horizons, mountains, plants, and trees.

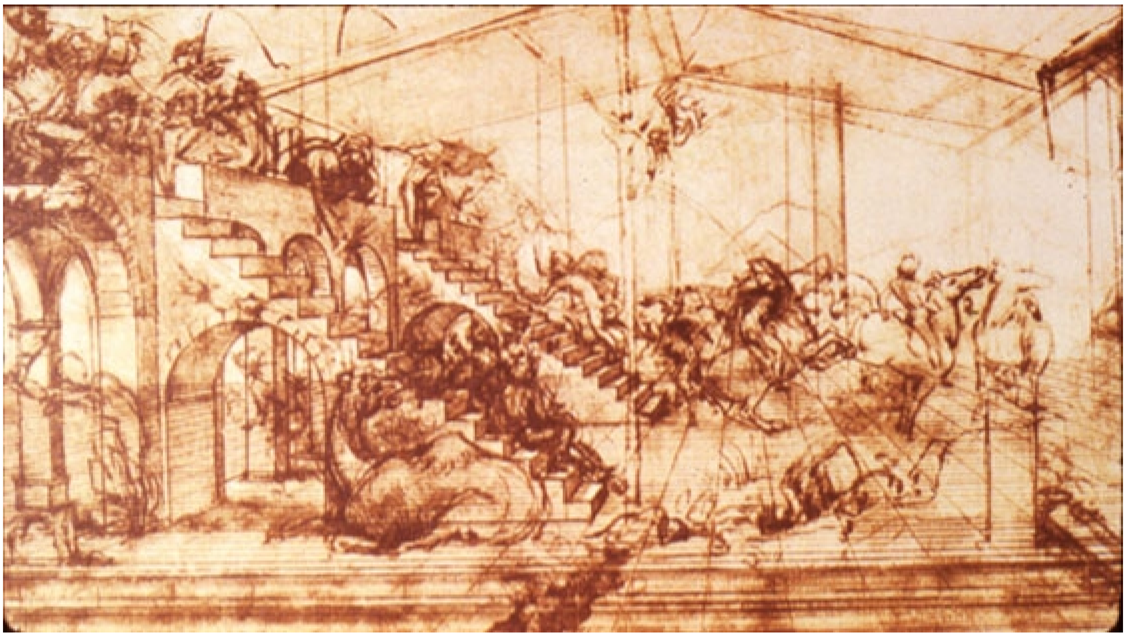
***Painters Practice**:* ethics, judging works, advice to young painters, the painter's life, the studio, aids, wall painting, invention and composition, allegories and emblems.

***The Science of Vision in Painting**:* properties of the eye and of light, color, perspective of size, color, and disappearance, and light and shade, or *chiaroscuro,* and linear perspective,

Perspective

Of all the topics in the *Tratatto*, we are mainly interested in perspective. Leonardo had said that *"perspective is the rein and rudder of painting.*" Invented by Brunelleschi, codified by Alberti and Piero, it was perfected by Leonardo.

Slide 15-25: The Adoration of the Magi, 1481 Reti p. 224



Leonardo's notes on linear perspective are apparently lost, but he made great use of perspective in his own paintings, such as this study of the unfinished *Adoration*. Note the strict Albertian grid on the pavement.

Slide 15-26: The Annunciation c. 1472 Ufizzi (Cat. # 1074)

His *Annunciation* shows a carefully worked out perspective framework. Incised lines beneath the paint on this wood panel show his construction. Note though that the Virgin's arm appears too long. Studies have shown that Leonardo departed from the correct perspective here for the sake of a more expressive gesture, a common practice in the Renaissance.

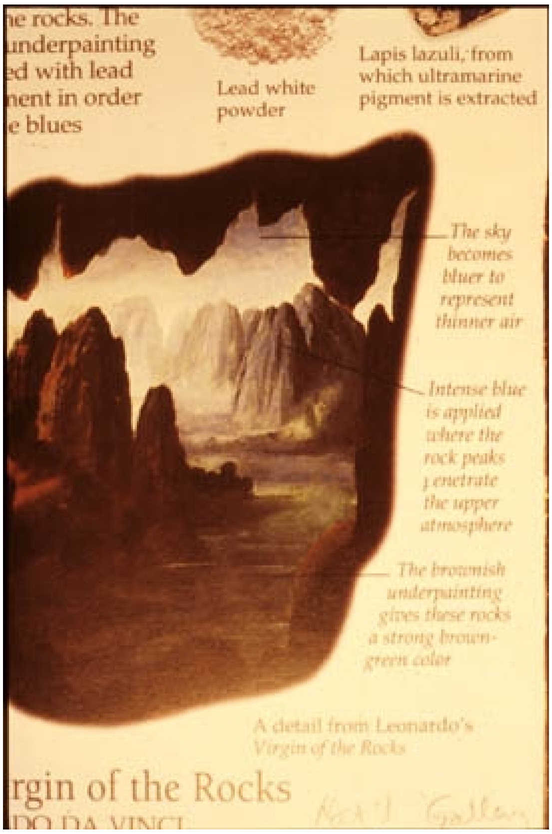
Slide 15-27: The Last Supper. c.1497. Mannering p. 42 (Cat. 3818)

Leonardo's most famous perspective painting, like all the other *Last Suppers*, this one is placed in a refectory or dining hall, here in the Convent of Santa Maria delle Grazie, Milan. The vanishing point is placed at Christ's right eye, where he dominates the foreground. Even Christ's arms reinforce the perspective, with his arms along the lines of the visual pyramid.

Atmospheric Perspective

Slide 15-28: c. 1508, Nat. Gall.

Slide 15-29: Detail Cole p. 28



Leonardo also describes another kind of perspective, now called atmospheric perspective. His writings on atmospheric perspective, the *perspective of disappearance*, have survived. For Kenneth Clark they anticipate the doctrines of impressionism. Distant objects appear smaller, less distinct, paler, and bluer, as seen in *The Virgin of the Rocks*.

Summary

To summarize, we have three Renaissance men, linked to each other as math teacher to pupil;

Piero taught Luca; Luca taught Leonardo.

They are also linked to each other by the platonic solids: Piero wrote the book on them, Luca stole and published it, Leonardo illustrated it.

In these last two units we've gone from the Early Renaissance of Piero to the high Renaissance of Leonardo; In between stood Pacioli, at the turning point between two worlds. Much of Luca's mathematics was inseparable from religion; an expression of perfection, therefore the divine." But if *three* to Luca represented the trinity; to Leonardo *three was just three!*

Listen to Leonardo's rejection of magic: *Of all human opinions ... the most foolish is the belief in Necromancy, the sister of Alchemy ... because it brings forth nothing but what is like itself, that is, lies ... [it is] the guide of the stupid crowd which is constantly witness to the dazzling and endless effects of this art ...*

Leonardo was a contemporary of Copernicus (1473-1543), and a precursor of Galileo (1564-1642). Coming soon are Kepler (1571-1630), Descartes (1596-1650), Fermat (1601-65), Pascal (1623-65). According to Rohatyn, *"Leonardo's insistence on first-hand inquiry ... is one of the sparks that ignite the scientific revolution and bring us, for better or for worse, into the modern era."*

Bibliography

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Kemp, *Leonardo on Painting*

J.P. Richter

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Cole, p. 24, 25, 28

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Vasari, Chapter on Leonardo

Pedoe, Chap. 3, pp. 82-101

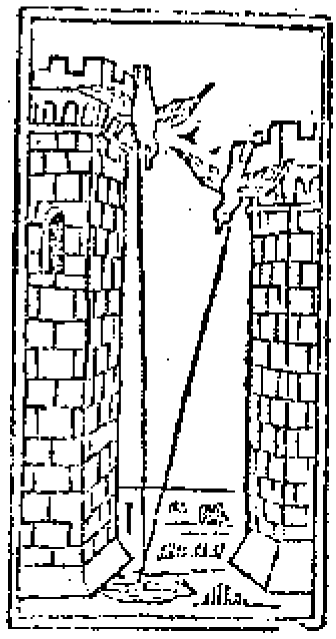
FAÇADE MEASUREMENT BY TRIGONOMETRY

Esono dua torri in uno piano che luna e alta 80 braccia et laltra e alta 90 braccia: et dal luna torre allaltra e 100 braccia: et intra queste dua torri e una fonte dacqua in tal luogho che movendosi due ucceglli uno diciascuna et volando dipari volo giungono alla detta fonte aun tracto. Vo sapere quanto lafonte sara presso a ciascuna torre

80——90——100
80 ✓ 90 ✓ 100 ✓
6400 8100 10000

8100
6400
1700
10000 | 200
11700
58 1/2

sara presso alla torre dell. 80 braccia a braccia 58 1/2 et lau.in 50 infino i 100 che ne 41 1/2 braccia sara presso a quella di 90 braccia



60. Surveying exercise. From Filippo Calandri. *De arimethrica* (Florence, 1491) p. c viii v.

Figure: Surveying Exercise. From Fillipo Calandri, *De arithmetica (Florence 1491).*
Baxandall p. 106

We are all familiar with the trigonometry textbook problem, *"The angle of elevation to the top of a building from a point 200 feet from ... Find the height of the building,"* and such methods are hardly new. Here we describe a trigonometric method than not only measures heights of points on a building, but widths and *depths* of those points. It will give the height, horizontal position, and depth, (*x, y*, and *z* coordinates) of each selected point.

This method was developed for the purpose of measuring Medieval and Renaissance structures in Italy, for research in the history of architecture. To measure a building, a historian is most likely to use a tape measure from scaffolding set up for that purpose, a direct but costly and laborious method. This method provides a more accurate and less expensive alternative.

Outline:

Reading

Background

324. The *radio astronomico* used to measure the width of a façade from Gemma's Frisius's *De Radio astronomico. . . .*, Antwerp, 1545.

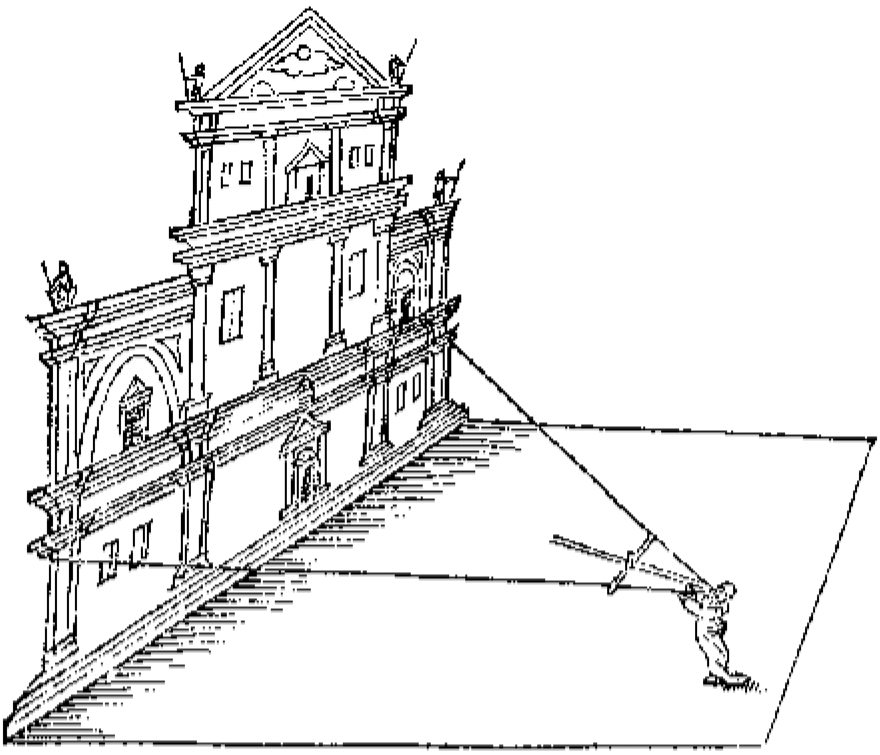


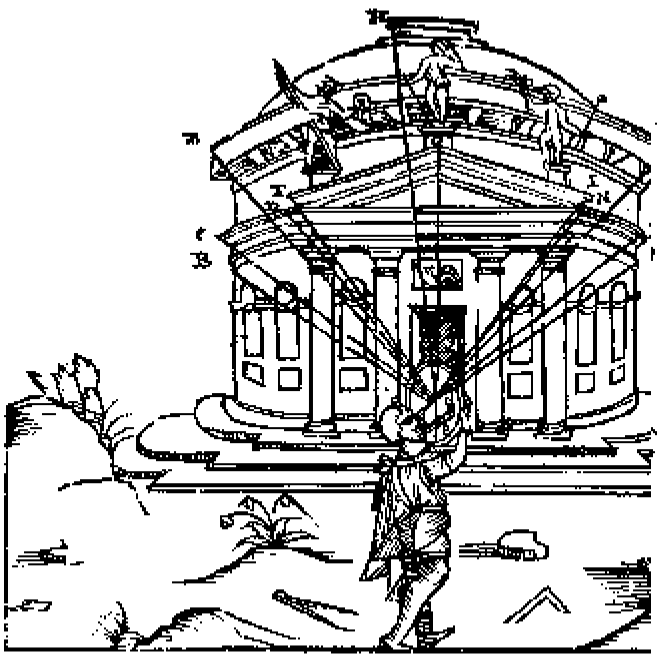
Figure: The *radio astronomico* used to measure the width of a façade. Frisius, 1545. From Kemp p. 169.

A literature search revealed few references to a trigonometric method. Martin Kemp, when talking about Fillipo Brunelleschi, the architect of the cupola of the Cathedral in Florence, says "On his first visit to Rome, as described in his biography, he made measured drawings of Roman buildings, using his understanding of standard surveying techniques 'to plot the elevations', using measurements 'from base to base' and simple calculations based on triangulation. The basis for such procedures would have been the 'abacus mathematics' he learnt as a boy." His source for this information is Antonio Manetti's Life of Brunelleschi." A search of Manetti's biography found reference to a visit to Rome, but no mention of his use of trigonometry to measure façades. In fact, there is some doubt expressed by the editor, Howard Saalman, that Brunelleschi ever went to Rome, and that this passage was added to enhance the stature of Manetti's subject.

Also according to Kemp, Leonardo recorded in the Codice Atlantico a cross-shaped measuring staff which he called the *bacolo* of Euclid, which was used to establish similar triangles. This instrument was perfected in the sixteenth century as the *radio astronomico* by the geographer and astronomer Gemma Frisius, who commends it for terrestrial as well as astronomical measurements.

Figure: An astrolabe used for surveying a building. From Cosimo Bartoli, *Modo di misurare*, Venice 1589. Kemp p. 168.

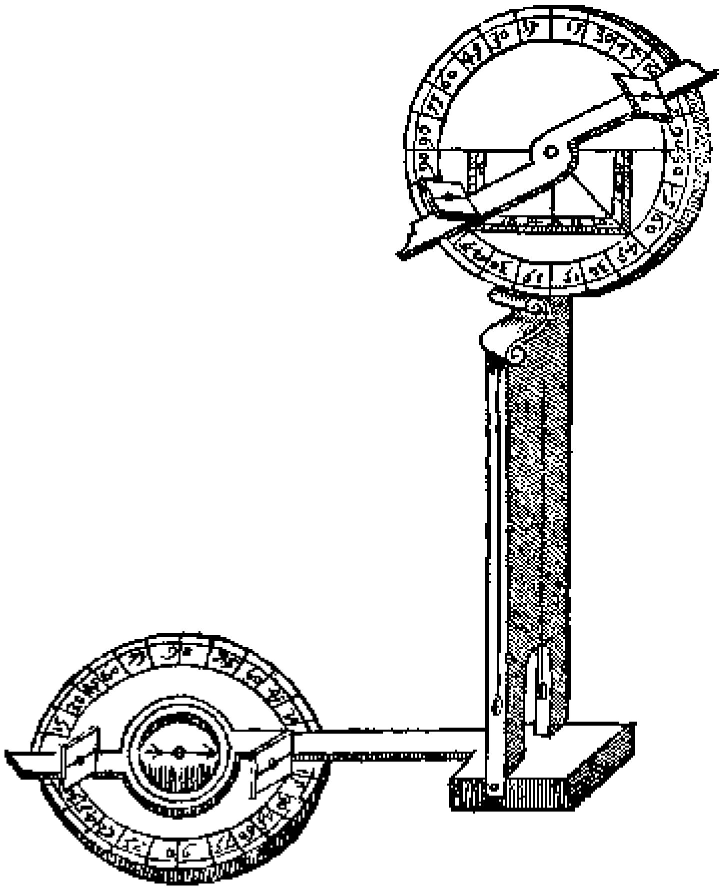
322. An astrolabe used for surveying a building, from Cosim
Modo di misurare. Venice, 1589.



Kemp speaks of mediaeval instruments of considerable elaboration and precision, most notably quadrants and astrolabes, which could be used for terrestrial mensuration, although this was not their prime function. Cosimo Bartoli shows an astrolabe being used for the measurement of a building.

Figure: A compound *bussola*, from Bartoli. Kemp p. 170,

326. A compound '*bussola*' for taking horizontal and vertical bearings, from Bartoli, *Del Modo di misurare*.

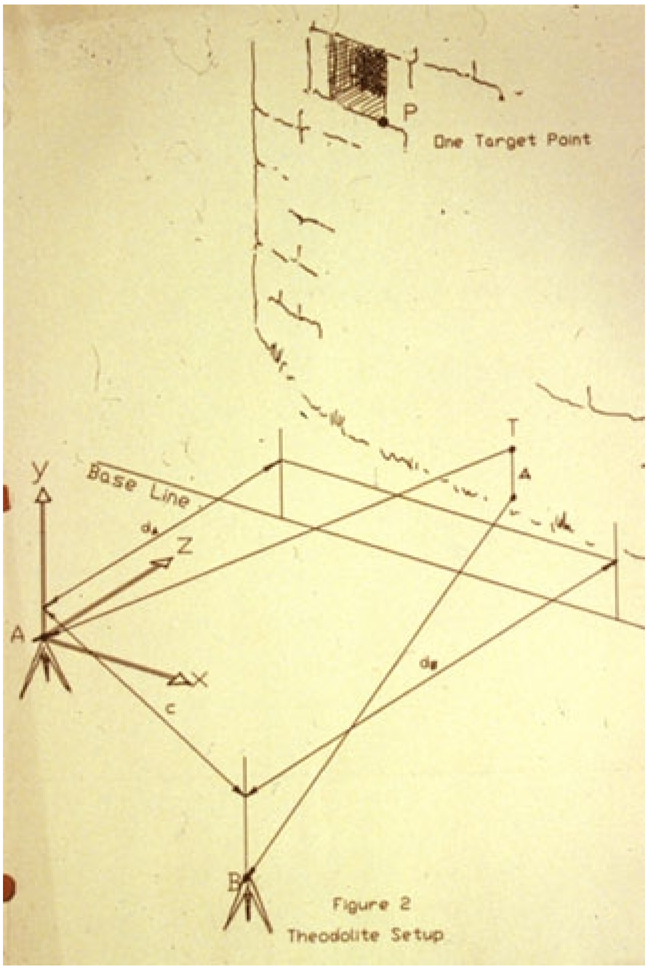


Raphael described a circular instrument used for his survey of ancient Rome, at the center of which is a compass, and a peripheral scale with arms that carry sight-vanes. A similar device called a *bussola* is described in Bartoli's book. The theodolite used for our present method is nothing but a more precise version of the bussola.

More recently, dimensioned drawings of façades have also been made by stereophotogrammatry, such as those for Independence Hall in Philadelphia. The Pantheon in Rome was surveyed by a method that used two electronic theodolites to simultaneously sight a point on the structure, their output being fed to a computer to give an instantaneous readout of coordinates.

The Method

Slide 18-2: Theodolite Setup



The equipment needed for this method is a surveyor's tape and a theodolite, an instrument for measurement of horizontal and vertical angles. It consists of a bubble level to establish the horizontal and vertical, a telescope that can rotate vertically in a mount that can also turn horizontally, with precise scales to read the angles.

To get the depth dimension, the procedure requires two theodolite setup positions, with a set of readings taken from each location. This second setup also provides a second set of numbers with which to check the first. This method will work with walls that are leaning out of plumb, have offsets, are curved, or have projecting elements like sills or cornices.

This procedure does *not* require the theodolite to be at the same height at each position, thus is suitable for sighting from *sloping ground*. Further, it is not required that the two theodolite positions be at the same distance from the wall.

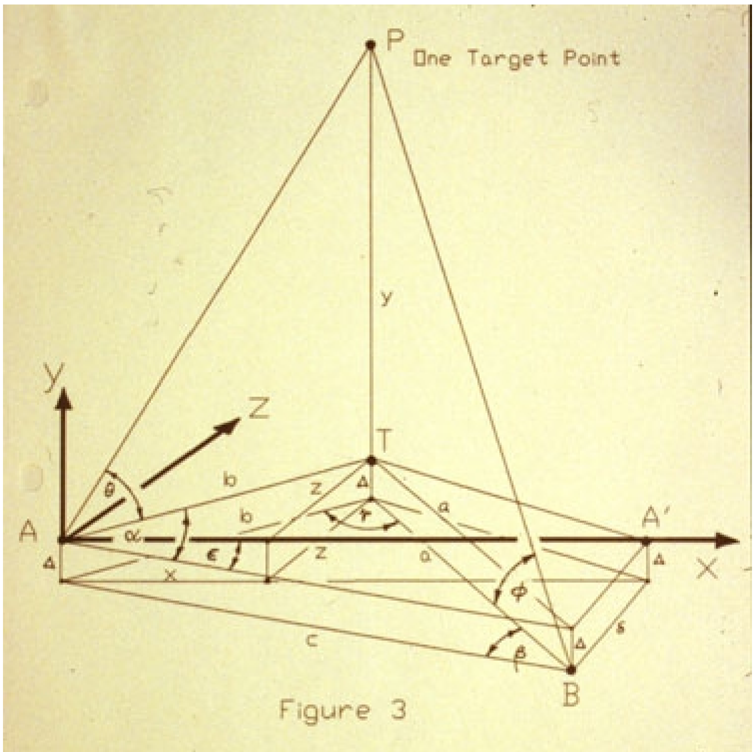
The procedure will give *two values* for each *y* coordinate, which are independent and can be averaged to give a final value.

Procedure

1. Study the façade. Take photos. Measure by manual taping whatever can be easily reached. Make a preliminary drawing. Choose and number the target points. Place adhesive targets on the wall, where possible.

2. Select or lay out a base line. The intersection of the façade and pavement makes a good base line, if it is straight and horizontal. Use a stretched cord if no suitable physical base line is available, as shown in the figure. The figure shows what is possibly the most difficult measuring situation, a curved building on sloping ground. Mark two theodolite setup points A and B on the ground or pavement, which can be at different heights and at different distances from the base line. Record their horizontal distance *c* apart and their horizontal distances d_A and d_B to the base line.
3. Set up and level the theodolite at location A. With the telescope horizontal, sight and mark a point T at any place on the wall also visible from location B.
4. Set a plumb line over the other theodolite location. Sight the plumb line with the theodolite and adjust the horizontal scale of the theodolite to read zero.

Slide 18-4: The Angles to be Measured



5. Sight each target. For each, record the horizontal angle α and the vertical angle θ.
6. After each target has been sighted, move the theodolite to the second location. With the telescope horizontal, sight a point R on the wall vertically in line with point T, found in step 3. Measure the vertical distance Δ from that point to T.
7. Repeat steps 4 and 5, recording the horizontal angle β and the vertical angle n for each target point.
8. Enter all measurements into the computer spreadsheet and print out the *x, y,* and *z* coordinate of each target point.
9. Make a final dimensioned drawing by hand or by use of a CADD program.

Derivation of Façade Equations

The equations that the spreadsheet uses to reduce the data are easily derived. Starting with the three original taped measurements,

c = Horizontal distance between theodolite locations.

d_A and d_B are the horizontal perpendicular distances from base line to theodolite locations.

Δ = Vertical offset between theodolites

From these we get

δ = Horizontal offset = $d_B - d_A$

ε = Angular offset = arcsin (δ/c)

L = distance AA' between A & B parallel to baseline = $\sqrt{c^2 - \delta^2}$

For each target P we have,

α = horizontal angle at A from B to target

θ = vertical angle at A from horizontal to target

β = horizontal angle at B from A to target

ϕ = vertical angle at B from horizontal to target.

Our coordinate axes will be as shown in the figures, with the origin at A, with the x axis parallel to the baseline and directed to the right, the y axis vertical and directed upwards, and the z axis perpendicular to the $x$and y axes, and directed towards the building. A simple translation of axes will later place the origin at any selected point, such as a corner of the building, and a rotation of axes can adjust the base line to be parallel to any selected line.

We now calculate the x coordinate of point P.

$$\gamma = 180 - \alpha - \beta$$
$$a = c \sin \alpha / \sin \gamma$$
$$b = c \sin \beta / \sin \gamma$$

Figure: Plan View

From the plan view we see that

$$\cos(\alpha - \varepsilon) = x / b$$

$$x = b \cos(\alpha - \varepsilon)$$

Next we find the y coordinate of point P. From position A:

$$\tan \theta = y / b$$

From position B:

$$y = b \tan \theta$$

$$\tan \phi = (y + \Delta) / a$$

$$y = a \tan \phi - \Delta$$

As mentioned, the values of y found from each setup position *are* independent. Next we find the z coordinate of point P.

$$\sin(\alpha - \varepsilon) = z / b$$

From which,

$$z = b \sin(\alpha - \varepsilon)$$

Field Test

Figure: Green Academic Center

The method was tested by taking measurements of the front of Green Academic Center at Vermont Technical College. The figure shows eleven target points, all visible from both theodolite locations. These were sighted using a Wild T2 theodolite, capable of a precision of about 0.2 seconds of arc. The baselines were taped three times using a standard surveyor's tape graduated in millimeters, and the readings averaged. The data was reduced using a spreadsheet. Some of the distances measured by theodolite were also taped, for comparison.

A pair of values was obtained for each y coordinate, corresponding to the two equations used for their calculation. These, of course, should be identical for each target point, and their difference gives us some measure of the precision of the method. Here we found a deviation from their average value of less than 0.2%.

Comparing points that are expected to be at the same height on the building, or at the same depth, or on the same vertical, we found differences less than a few millimeters. These deviations may represent inaccuracies in the measurements, or may represent actual differences in height of these points.

On the basis of this one test, it would appear that, with moderate care, accuracies within a few centimeters, or within 2%, are easily obtained. There is no theoretical limit to the accuracy of the method.

Measurements in Italy

Torre Bernarda

Slide 18-6: Torre Bernarda

The first real use of the method was on the western façade of the *Torre Bernarda*, a Medieval tower in the town of Fucecchio, near Florence.

Slide 18-8: Sighting the Torre

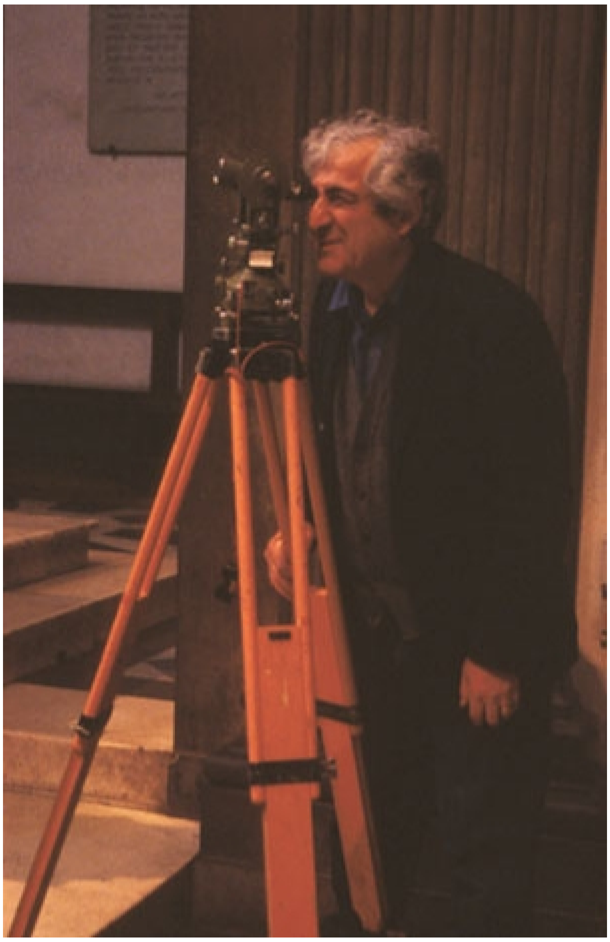
Since the terrain did not enable us to establish a base line parallel to the façade, a modified procedure was used. The same equations used before apply here, but with the horizontal offset and the angular offset both equal to zero. Then by a simple translation of axes, the origin was placed at a convenient point on the façade, and a rotation of axes put the x axis in the plane of the façade. Accuracies obtained were of the same order as for the VTC measurements.

Medici Chapel

Slide 18-10: Medici Chapel Interior

A second use for the method was measurement of the interior of Michelangelo's Medici Chapel in Florence. This was part of a project to deduce the systems of proportions Michelangelo may have used in its design.

Due to the darkness of the interior, we used a laser pointer to mark the approximate position of a measuring point, and illuminated that feature with a flashlight.



Laurentian Library

Figure: Entrance to the Laurentian Library, from the Vestibule

A third project was the doorway to the Laurentian Library in Florence, again to investigate what proportions Michelangelo may have used. Here the door is shown as seen from the vestibule, just outside the library itself.

Summary

In conclusion. we have here a fast, inexpensive, low-tech tool for measuring façades capable of giving accuracies of less than 1%, which, for good measure has roots firmly planted in the history of architecture.

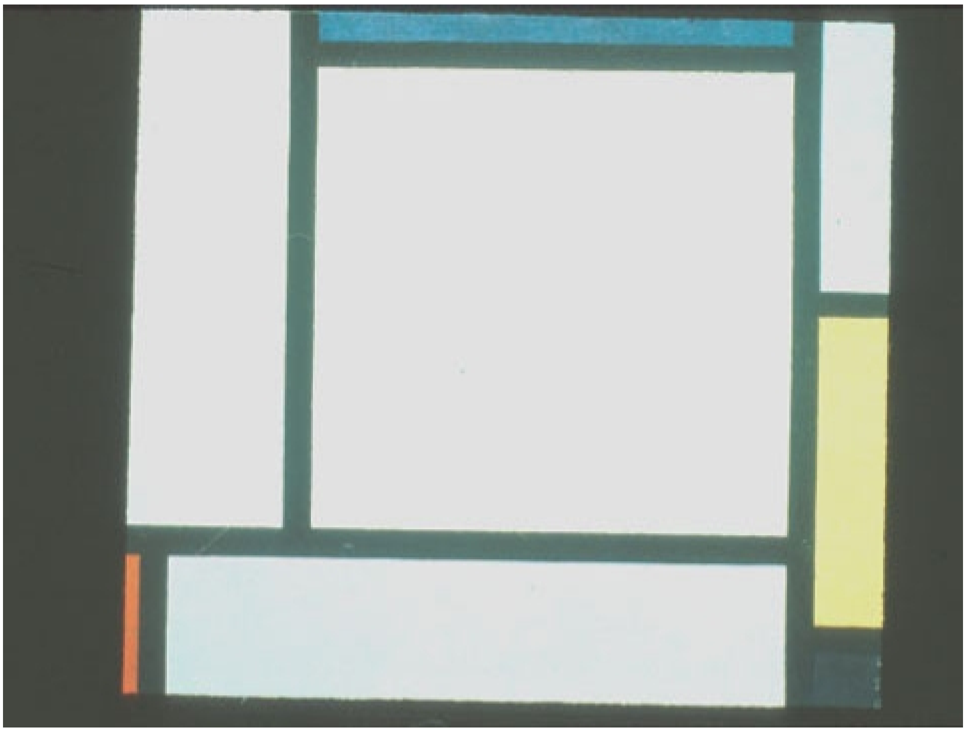
Reading

Façade Measurement by Trigonometry. Paul Calter, from *Nexus: Architecture and Mathematics*, Kim Williams, editor, 1996.

Martin Kemp, *The Science of Art.* New Haven: Yale U. Press, 1990

EARLY TWENTIETH-CENTURY

GEOMETRIC ART



Slide 19-12: Mondrian: ***Composition,*** **1933**

In this unit we will show some of the roots of geometric abstraction in the Twentieth Century, and present an overview of some of the early movements.

Up to now we've seen paintings, sculpture, and architecture that had geometric elements: geometric figures, particular ratios like the golden ratio and the musical ratios, and where space was organized by the geometric perspective construction. But now, we have paintings and sculpture that is **totally geometric.**

We've already seen decorative ornamentation that was completely geometric, with no recognizable elements. But these are not decorative ornaments; this is high-class art found in museums and galleries. This kind of art is called ***geometric abstraction.***

Geometric Abstraction

551 Constantin Brancusi. *The Kiss.* 1908. Limestone, height 23" (58 cm). Philadelphia Museum of Art (Louise and Walter Arensberg Collection).

OT 1: Brancusi: *The Kiss*, 1908

Three Main Currents

The art historian H.W. Janson sees *three main currents* in art, beginning near the start of the century, *Expression, Fantasy, and Abstraction.*

Expression deals with feelings, and the concern with the human community. These artists stress their *emotional* attitude towards the world. Imagination and fantasy explore the the labyrinth of the mind, and Abstraction stresses order and the *formal structure* of the work.

The best works have all three:
Without feeling we are unmoved.
Without imagination, we are bored.
Without order, we see chaos

In each of these, works have been made ranging from **realistic** to **non-representational (non-objective).** In this unit we are going to examine non-representational abstraction; in particular, **geometric abstraction.**

What is Geometric Abstraction

Starting around the turn of the century there was a complete rejection of literary academic art - artists didn't want to just make a copy of a real object - the newly-invented camera could do that faster and better. They didn't want to make an an illustration for a story. They rejected Alberti's *istoria* or history painting in which part of the painting's power came from the story it illustrated. The phrase *storybook realism* became a term of derision

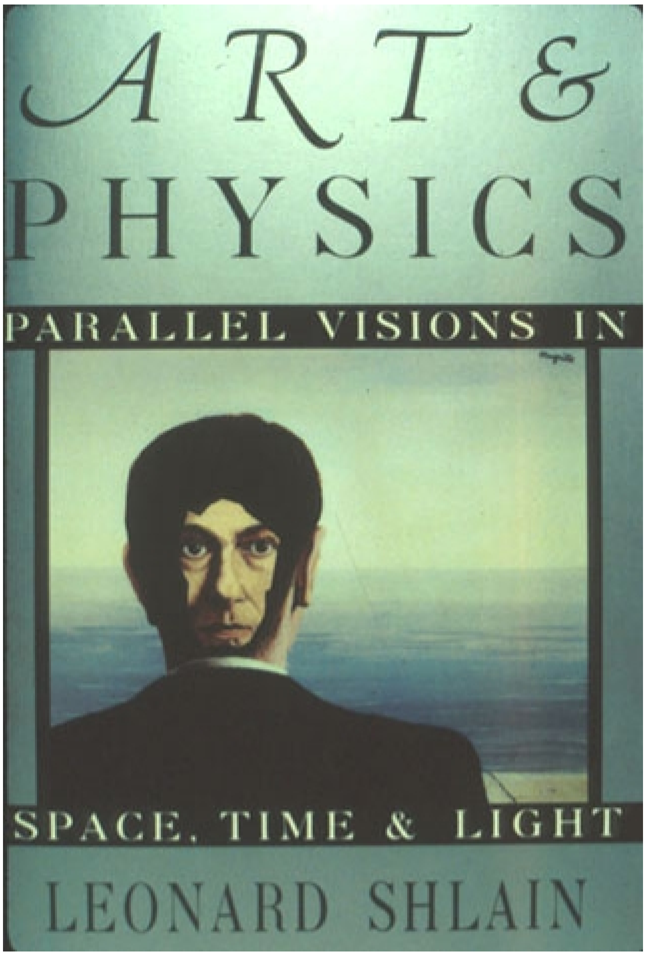
The idea now was that one doesn't paint about *anything* - one just *paints.*

The old visions were worn out. Artists wanted to create something that did not exist before, to see the world in new ways, to talk about their inner world, to grapple with large ideas that were universal and utopian.

Abstract art is an attempt to analyze and simplify what we see, to pick and choose. But a work can be abstract, like Brancusi's *Kiss*, and still be representational, while the Mondrian shown at the start of this unit is completely non-representational.

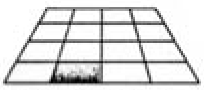
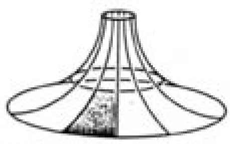
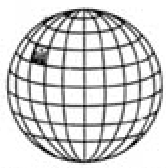
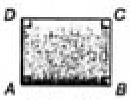
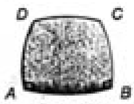
Let's try to trace the steps that took us from the impressionist landscapes of the late 19th century to total geometric abstractions of the twentieth.

The Scientific Revolution



Slide 19-1: Shlain: ***Art and Physics, cover***

TABLE 7.6 Comparison of Major Two-Dimensional Geometries

Euclidean geometry	*Hyperbolic geometry*	*Elliptic geometry*
Euclid (about 300 B.C.)	*Gauss, Bolyai, Lobachevski* (about 1830)	*Riemann* (about 1850)
Given a point not on a line, there is one and only one line through the point and parallel to the given line.	Given a point not on a line, there are an infinite number of lines through the point that do not intersect the given line.	There are no parallels.
Geometry on a plane	Geometry on a pseudosphere	Geometry on a sphere
		
The sum of the angles of a triangle is 180°.	The sum of the angles of a triangle is less than 180°.	The sum of the angles of a triangle is more than 180°.
 D C A B	 D C A B	 D C A B
$D = 90°$	$D < 90°$	$D > 90°$
Lines are infinitely long.	Lines are infinitely long.	Lines are finite in length.

OT 2: Comparison of Major Two-Dimensional Geometries. Smith, *The Nature of Mathematics*, p. 501

Art isn't created in a vacuum. It usually reflects what is going on elsewhere in a culture.

One thing that was happening at the start of our century was a scientific revolution

It appears that at the start of the our century science and art once more were ready for new concepts of space and time. Some new space concepts came in geometry with the **Non-Euclidean Geometries** of Bolyai, Lobachevski, and Riemann in the mid ninteenth century. New time concepts came with **Einstein's theories, the** special theory of relativity, 1905 and the general theory of relativity, 1915.

Influence of photography

About 1883, American inventor George Eastman produced a film consisting of a long paper strip coated with a sensitive emulsion. In 1889 Eastman produced the first transparent, flexible film support, in the form of ribbons of cellulose nitrate. The invention of roll film marked the end of the early photographic era and the beginning of a period during which thousands of amateur photographers became interested in the new process.

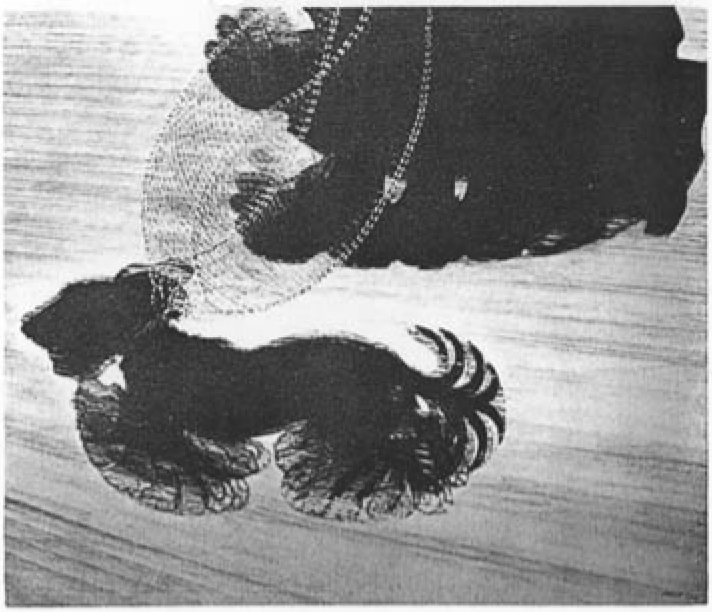
In the early 20th century, commercial photography grew rapidly, and improvements in black-and-white photography opened the field to individuals lacking the time and skill to master the earlier, more complicated processes. The first commercial color-film materials, coated glass plates called *Autochromes Lumière* -- after the process developed by French inventors Auguste and Louis Lumière -- became available in 1907. During this period, color photographs were produced with the three-exposure camera.

This prompted a move away from **representation** and **realism** and towards abstraction. If you can capture a scene with the snap of the shutter then why sit there for hours copying it in paint.

Muybridge, Eadweard (1830-1904), English-American photographer and motion picture pioneer, known for his photographs of animals and people in motion. In 1877 he demonstrated through photographs that when a horse runs, there is a moment when all of the animal's feet are off the ground, and that the feet are tucked beneath the animal at that moment. In 1881 he invented the zoopraxiscope, a device by which he reproduced on a screen horse races, the flights of birds, and athletic contests. He wrote *The Horse in Motion* (1878) and *Animal Locomotion* (11 vol., including 100,000 photographic plates, 1887). Portions of the latter work were published under the titles *Animals in Motion* and *The Human Figure in Motion* (1901).

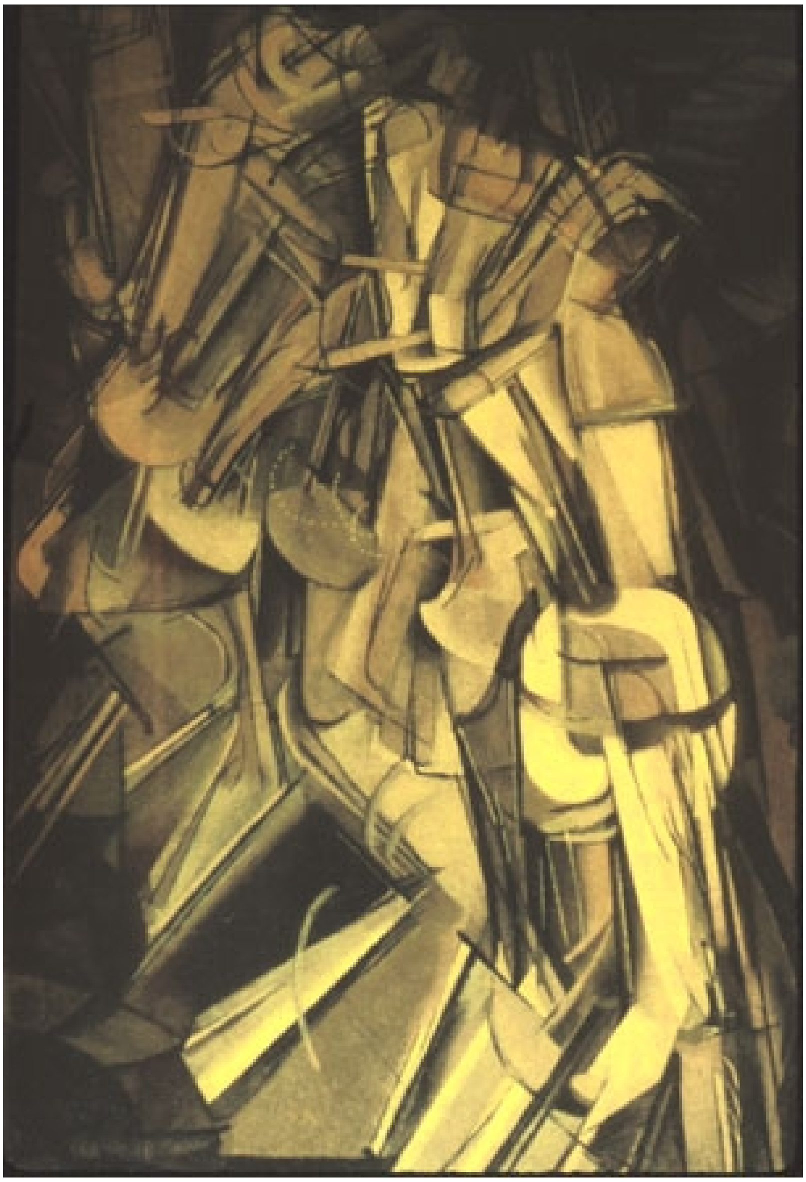
Through Pictures such as these people became used to seeing figures or parts of figures at the same time

Simultaneity



584. Balla: *Leash in Motion*, 1912. 35⅝ x 43¼". Collection A. Conger Goodyear, New York.

OT 4: Balla: *Leash in Motion,* 1912



Slide 19-2: Duchamp: ***Nude Descending a Staircase,*** **1912**(Jan 776)

Some works that show a time element include those by the Italian Futurists, who sought means of expression compatible with the modern industrial world, and *Nude Descending a Staircase*, nicknamed "Explosion in a Shingle Factory." In the Armory Show in 1913, it needed to be protected by guards.

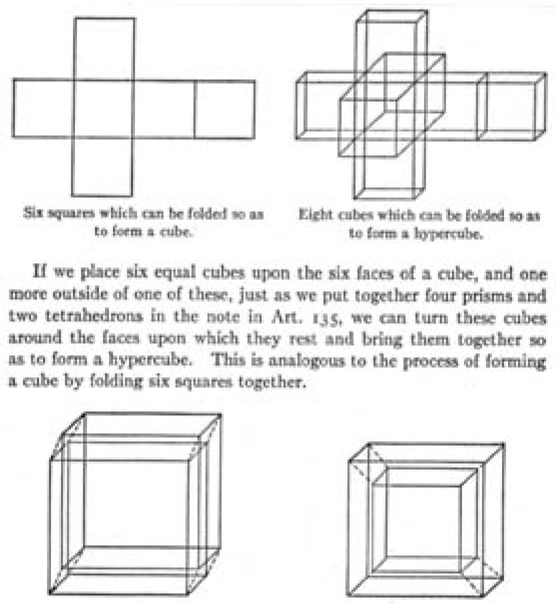
The Fourth Dimension

There seemed to be a lot of fascination with the **fourth dimension** early in this century, but it meant different things to different people:

- **Time** is often considered the fourth dimension in the space-time continuum with three space dimensions and one time dimension.
- **Color** has been described as a dimension.
- For some artists the fourth dimension appears to have been a metaphor for **liberation from the conventions of linear perspective.**

- To some philosophers it was a **physical reality** to which we have limited access.
- To many mathematicians, the fourth dimension simply means **an abstract space** described in terms of four mutually perpendicular axes.

Hypercube



Six squares which can be folded so as to form a cube.

Eight cubes which can be folded so as to form a hypercube.

If we place six equal cubes upon the six faces of a cube, and one more outside of one of these, just as we put together four prisms and two tetrahedrons in the note in Art. 135, we can turn these cubes around the faces upon which they rest and bring them together so as to form a hypercube. This is analogous to the process of forming a cube by folding six squares together.

OT 5: Hypercube, Manning p. 240

One four-dimensional object is the hypercube, shown here in an illustration from a textbook on geometries of higher dimensions. It shows how a hypercube can be folded from 8 cubes, just as a regular cube can be folded from 6 squares.

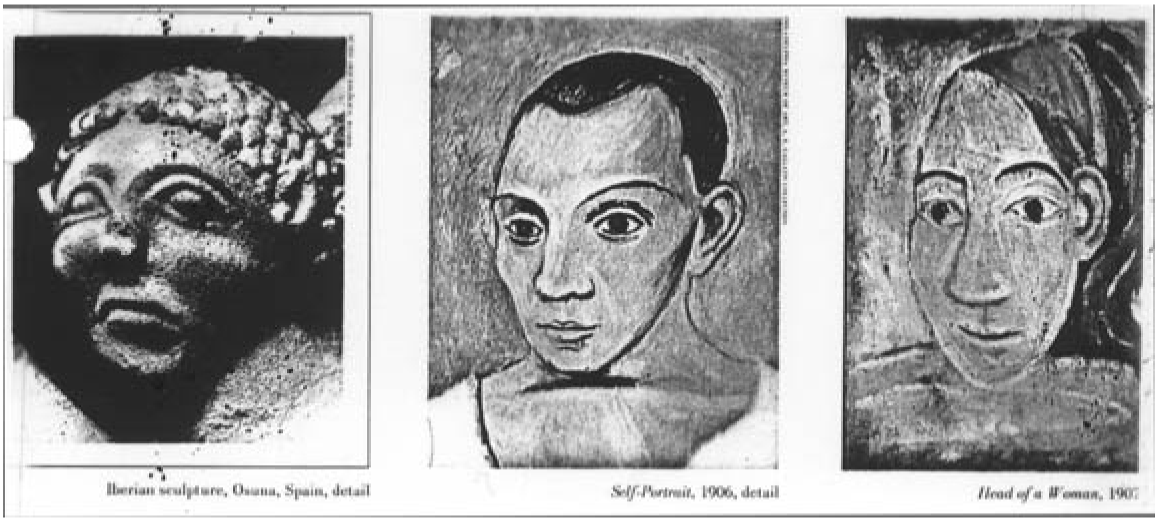
Fig. 14. Salvador Dali, *Corpus Hypercubicus* (The Crucifixion), oil on canvas, $76\frac{1}{2} \times 48\frac{3}{4}$ in, 1955. (Metropolitan Museum of Art, gift of Chester Dale Collection)

OT 6: Salvador Dali: *Corpus Hypercubicus*, 1955

Here the artist Salvador Dali has used an unfolded hypercube as a cross.

Cubism

Cubism's Two Parents



OT 7: African mask, Iberian sculpture

Geometric art has its roots in cubism. Cubism, on the other hand, was born of two parents: primitive art and Cezanne.

Cezanne Cubism

Slide 19-3: Cezanne: Rocky Landscape at Aix, 1887. (Harris p. 67)

In fact, the first stage of cubism is even called Cezanne Cubism. Note that the areas of the painting are becoming simplified and more geometric.

Braque

Slide 19-4: Houses Near l'Estaque. George Braque 1908. Cat #2578, p. 158 (Wolfe p. 8)

The very name *Cubism* came from this painting. When the critic Vauxcelles saw this painting he said the houses "look like a bunch of little cubes." He meant the comment to be insulting but the name Cubism stuck.

Kinds of Cubism

Cubism is sometimes broken down into three kinds by art historians:

Facet, Collage, and Analytical

This first stage, Cezanne cubism, is also called *facet* cubism, because of the different planes or facets. The stage called analytical cubism is typified by two or more views of a subject being given at once.

Analytical cubism

562. METZINGER: *Tea Time*, 1911. Panel, 29⅞ x 27⅜". Philadelphia Museum of Art, Arensberg Collection.

OT 3: Metzinger: *Tea Time,* 1911 "Mona Lisa with a Teaspoon."

These new ways of looking at the world may have influenced artists to introduce *time* into their works, as in *Tea Time.* Note that the face is both profile and front view, and the cup is both side and top view. It shows ***simultaneity***, as if one were walking around the scene. This is also an **attack on perspective**, which gives the scene from a single viewpoint.

Loss of Perspective

In a revolution old ideas are tossed out. We've already scrapped representation and storybook realism at the start of our century. Another discarded idea was perspective.

Perspective was used extensively up to the end of nineteenth century. But now the artist was saying, *This is a painting, not a window. This painting is about itself, not about something out there.*

Flatness

Cubism, for the first time since the Renaissance, dispensed with the need to produce an accurate copy of external reality. It acknowledged that a painting is a flat surface on which colored patches of pigment are arranged.

Constructivism

Collage cubism

Slide 19-5: Picasso: *Still life with chair caning.* 1911-12 (Janswon 771)

Braque was one of the pioneers of Cubism, but Picasso is credited as being the real creator of abstraction.

Up to now, all sculpture carved or modelled from a monolithic lump of material. But another branch of Cubism changed all that. As perspective was dropped images moved forward to the picture plane. Some images went even further and invaded the viewer's space, when Picasso and Braque invented the next stage of cubism, called *collage* cubism.

Collage = paste-up. Stuff glued on, paint added later.

This too, was something new in the history of art. Thick pictures are nothing new, recall Donatello's *Feast of Herod*. But collage is different from relief carving. It comes *out* from the picture plane instead of being carved in. It is built up.

Constructions

OT 8: Picasso: *Guitar*, 1912 and Violin, 1913-14

Collage was followed by Picasso's and Braque's openwork sculpture. Led to many sculptures that were built up, piece by piece, called constructivism.

Negative Space

Slide 19-13 Brancusi: ***Bird in Space,*****1928** (Janson 818)

Of course, monolithic sculpture didn't dissapear.

Slide 19-7: Henry Moore, *Recumbent Figure*, 1938. Janson p. 826

But now the open spaces, or **negative space**, played a more important role than before, and many sculptures had holes.

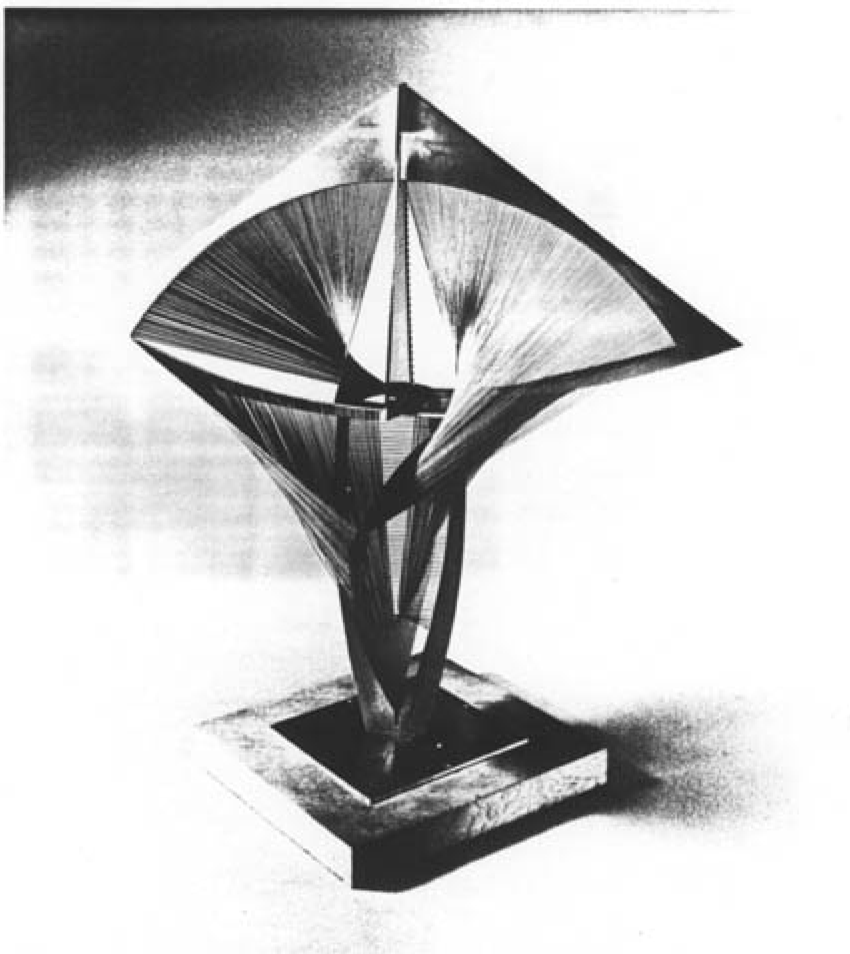
Russian Constructivism

Slide 19-8: Antoine Pevsner: ***Torso,*** **c. 1925**(Dab. 94)

Constructivism popular with Russian artists, like Pevsner and Gabo. But Russian Constructivism got tangled up with Communist doctrine, the role of the artist-engineer, the production of utilitarian objects, the rejection of painting as obsolete, and so forth, and had run its course by 1930.

Strings

Gabo, Naum
Torsion—Bronze Variation. 1963
Gold-plated bronze and stainless-steel springs
24½ × 28" (62 × 71 cm)



OT 9: Naum Gabo: *Torsion-Bronze Variation*, 1963

Picasso's *Guitar* and *Violin*, being stringed instruments, had, of course, strings. Other artists also used wire or rods in their works.

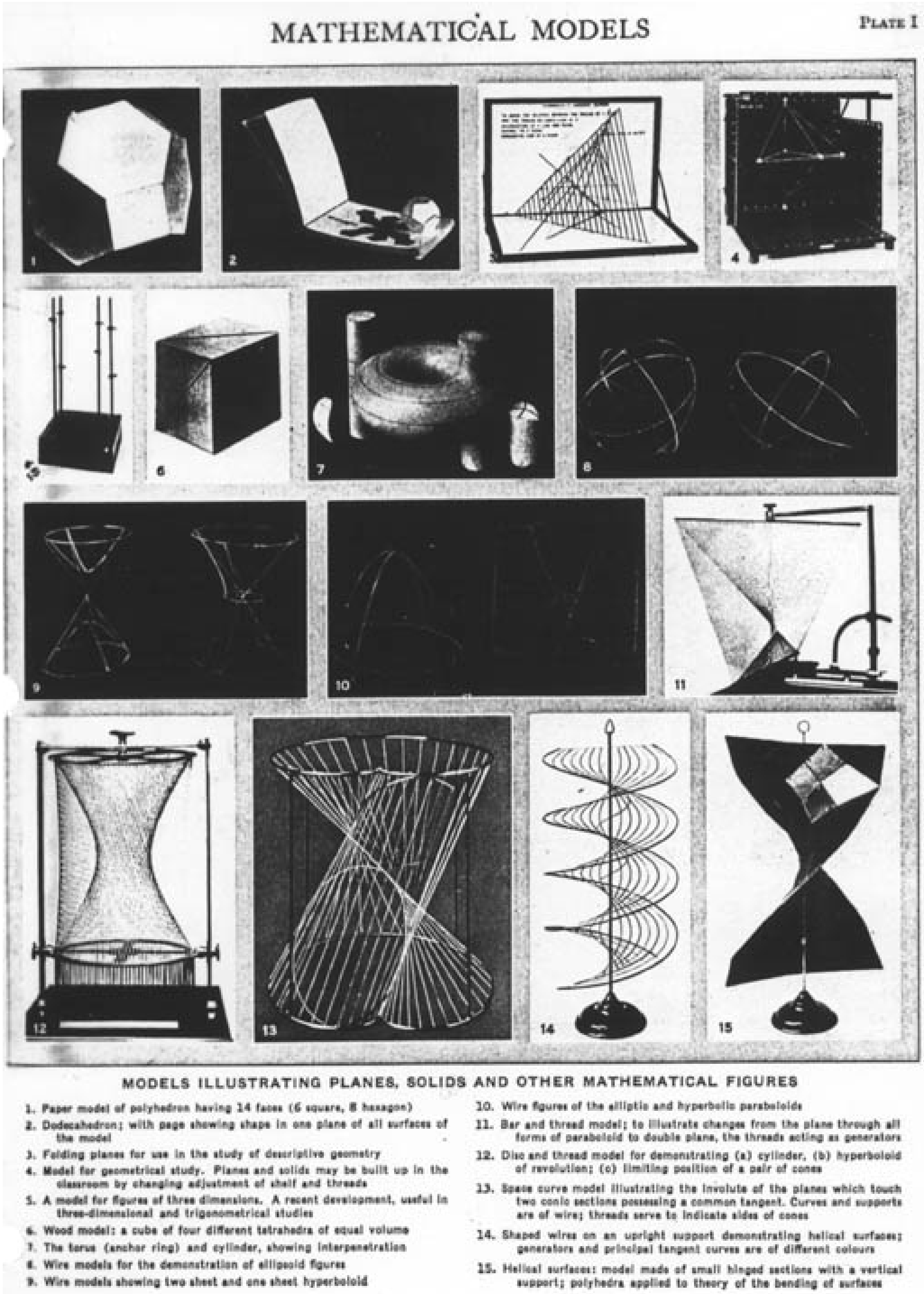

MATHEMATICAL MODELS

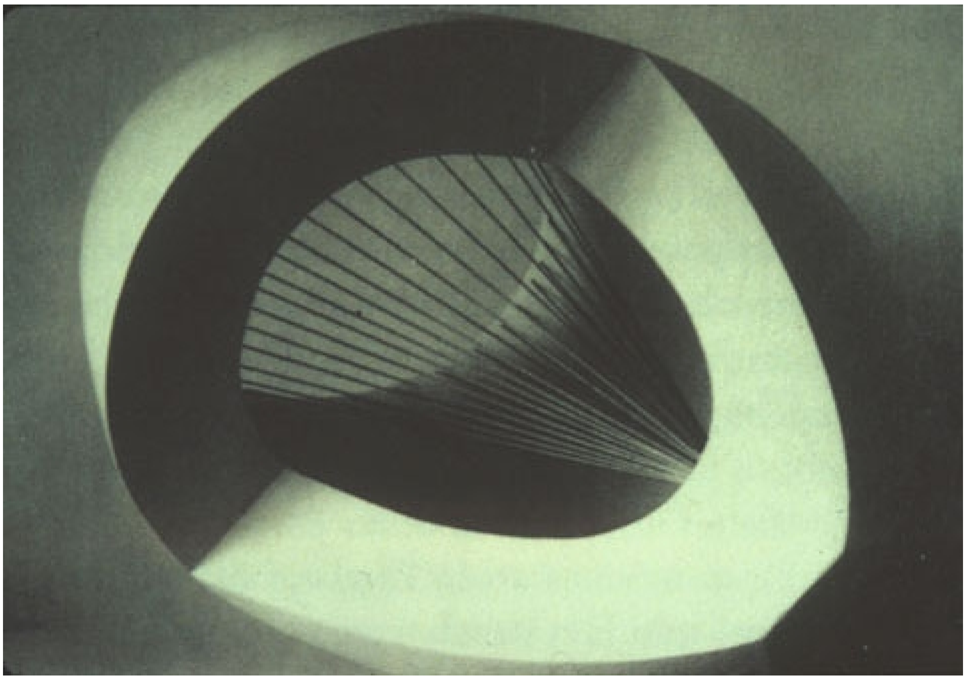
PLATE I

MODELS ILLUSTRATING PLANES, SOLIDS AND OTHER MATHEMATICAL FIGURES

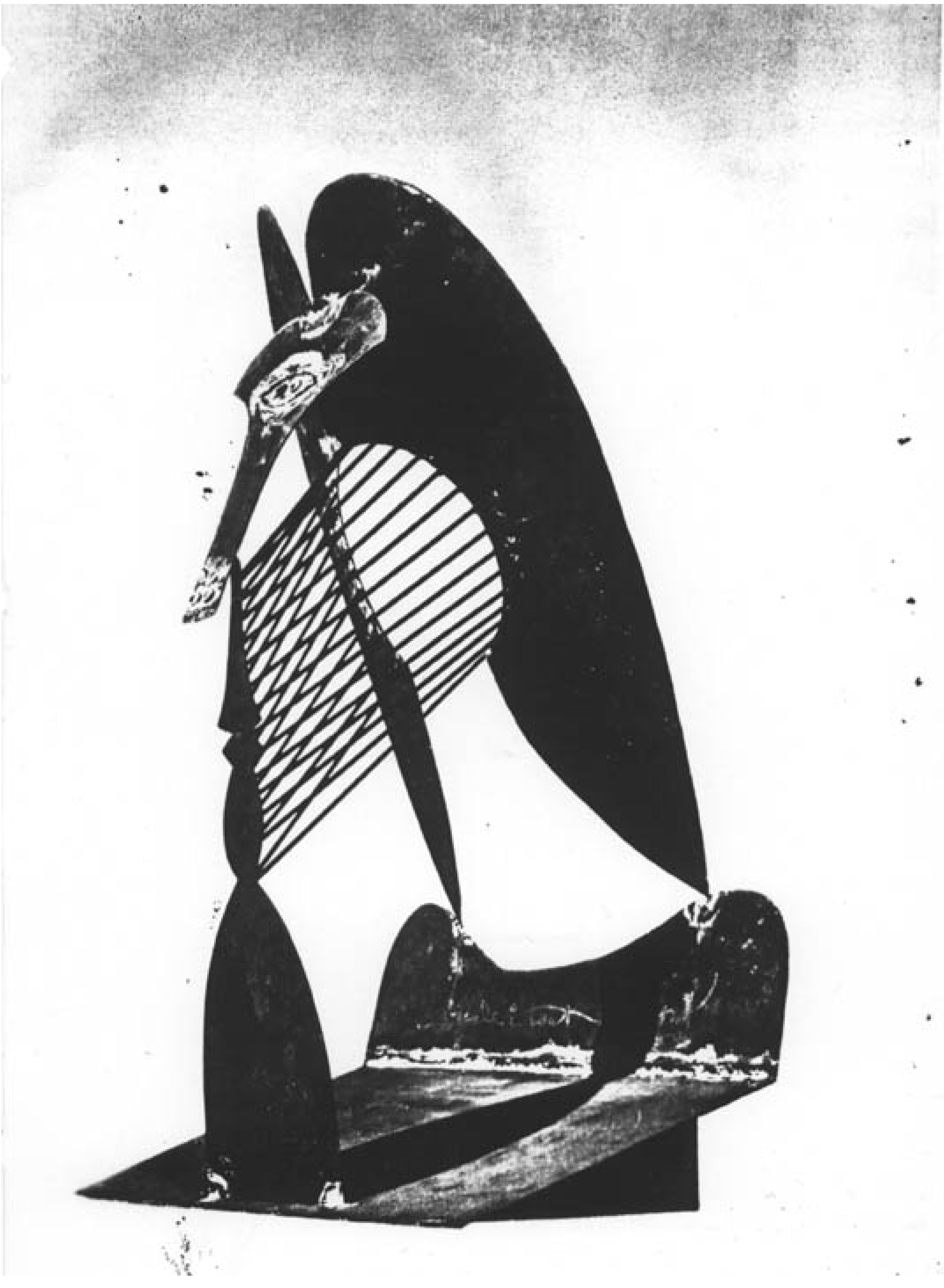
1. Paper model of polyhedron having 14 faces (6 square, 8 hexagon)
2. Dodecahedron; with page showing shape in one plane of all surfaces of the model
3. Folding planes for use in the study of descriptive geometry
4. Model for geometrical study. Planes and solids may be built up in the classroom by changing adjustment of shelf and threads
5. A model for figures of three dimensions. A recent development, useful in three-dimensional and trigonometrical studies
6. Wood model: a cube of four different tetrahedra of equal volume
7. The torus (anchor ring) and cylinder, showing interpenetration
8. Wire models for the demonstration of ellipsoid figures
9. Wire models showing two sheet and one sheet hyperboloid
10. Wire figures of the elliptic and hyperbolic paraboloids
11. Bar and thread model; to illustrate changes from the plane through all forms of paraboloid to double plane, the threads acting as generators
12. Disc and thread model for demonstrating (a) cylinder, (b) hyperboloid of revolution; (c) limiting position of a pair of cones
13. Space curve model illustrating the involute of the planes which touch two conic sections possessing a common tangent. Curves and supports are of wire; threads serve to indicate sides of cones
14. Shaped wires on an upright support demonstrating helical surfaces; generators and principal tangent curves are of different colours
15. Helical surfaces: model made of small hinged sections with a vertical support; polyhedra applied to theory of the bending of surfaces

OT 10: Encyclopedia Brittanica: ***Mathematical Models***

Henry Moore says he introduced strings in 1937 after seeing mathematical models in the science museum in London, where volumes of solids were indicated by strings threaded between geometric figures.



Slide 19-9: Barbara Hepworth: *Sculpture with Color*, 1943. Jan 826



OT 12: Picasso: *Chicago Civic Center sculpture*, 1967

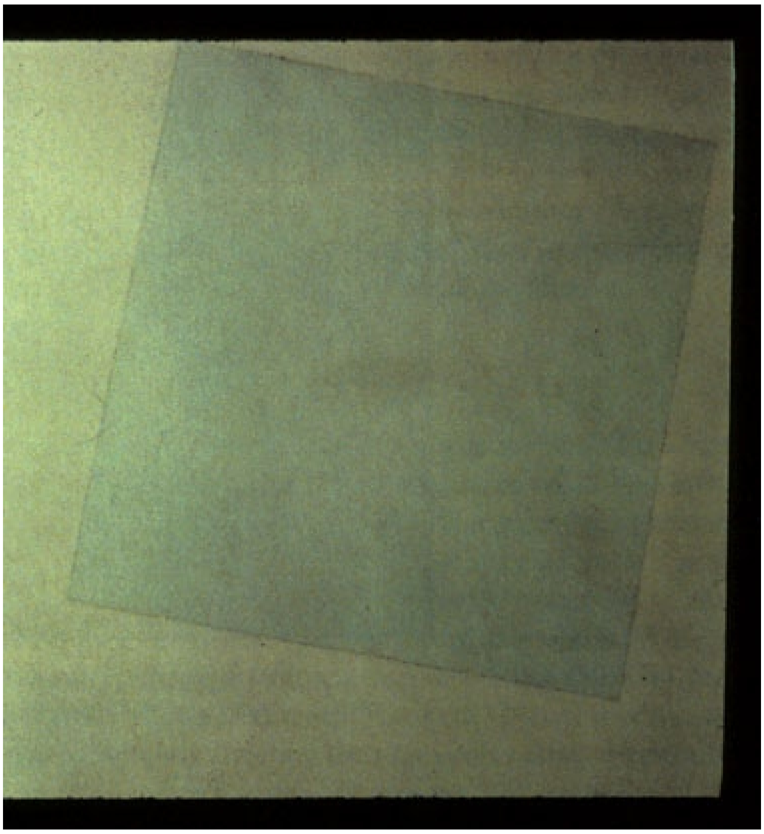
This is one of Picasso's later constructions. He did not identify it.

Suprematism

Slide 19-10: Kasimir Malevich, *Samovar,* c. 1913(Dab 38)

In addition to the artistic revolution of cubism and the scientific revolution of relativity there were political upheavals like the first world war. Following the war was the Russian revolution.

Malevich



Slide 19-11: Kasimir Malevich, *White on White* 1918(Jan 774)

Malevich's early paintings, like *Samovar,* were cubist in style. Then he started searching for a means of expression compatible to modern values, to create an art for the new industrial Communist state. He and others felt that the older art had lost relevance in a society changed by industrial technology, social upheaval, and Einstein's physics. Built on cubism and Futurism, it led to a movement called **Suprematism**.

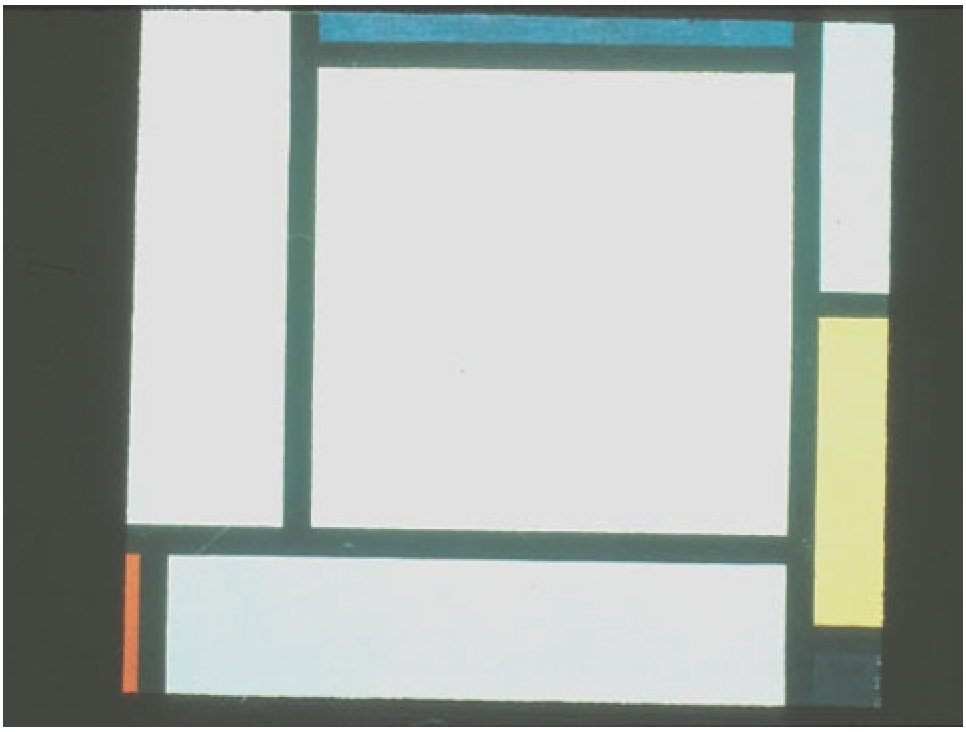
Suprematist paintings featured austere compositions, flat color, unstructured space, and geometric forms. Pure art to symbolize order and harmony of the new age. Its philosophical justification came in 1908 from a German philosopher Wilhelm Worringer:

". . . the urge to abstraction stands at the beginning of every art and in the case of certain peoples remains the dominant tendency . . . this urge is best served through pure geometric abstraction . . . free of all external connections with the world."

You can't be more disconnected than in this painting! Malevich had painted *Black Square on White Ground*, 1913, which he felt was the supreme suprematist composition, then *White on White* in 1918. *White on White* was the ultimate painting. The problem is, once you've painted the perfect painting, what do you do for an encore? Malevich quit painting and turned to architecture.

Before long this geometric art was rejected by the very audience for which intended. Russia turned to Socialist Realism, where muscular men in overalls and buxom women with babushkas are shown confidently building the new communist state.

De Stijl & Mondrian (1872-1944)



Slide 19-12: Mondrian: *Composition,* 1933

Piet Mondrian's early works show influence of Cubism, but gradually his paintings become more abstract. Annoyed at cubists for reversing course away from geometric abstraction, he took abstraction to its ultimate conclusion by creating total geometric abstractions. His paintings features:

simple geometrical forms

no natural forms

flat picture plane. No perspective

pure primary colors

rectangular grid

rigorous composition

De Stijl (the style) started 1917, at end of WWI, when the need for a new order was sharply felt. Like Suprematism, it too had sense of social destiny, and was very utopian. Its goals were, as for Suprematists, to create art that is a counterpart to the harmonious relations in the ideal society. De Stijl is considered the purest and most ideal of the movements.

Summary

We have seen how the new geometries of the nineteenth centuries, the scientific revolution of the early twentieth century, and the invention of photography have contributed to the radical changes in art of the early twentieth century.

We've traced some of these influences in early art movements, including Futurism, Cubism, Constructivism, Suprematism, and De Stijl. We've seen pictures become totally flat, many become totally abstract, many totally geometric, and after 500 years of development, perspective was scrapped. In a later unit, we'll continue following this thread into the late Twentieth Century