Quantum field theory and time machines

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We analyze the ''*F*-locality condition'' (proposed by Kay to be a mathematical implementation of a philosophical bias related to the equivalence principle, which we call it the ''*GH*-equivalence principle''), which is often used to build a generalization of quantum field theory to nonglobally hyperbolic spacetimes. In particular we argue that the theorem proved by Kay, Radzikowski, and Wald to the effect that time machines with compactly generated Cauchy horizons are incompatible with the *F*-locality condition actually does not support the ''chronology protection conjecture,'' but rather testifies that the *F*-locality condition must be modified or abandoned. We also show that this condition imposes a severe restriction on the geometry of the world (it is just this restriction that comes into conflict with the existence of a time machine), which does not follow from the above mentioned philosophical bias. So, one need not sacrifice the *GH*-equivalence principle to ''amend'' the *F*-locality condition. As an example we consider a particular modification, the ''*MF*-locality condition.'' The theory obtained by replacing the *F*-locality condition with the MF-locality condition possesses a few attractive features. One of them is that it is consistent with both locality and the existence of time machines. [S0556-2821(98)00124-6]

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I. INTRODUCTION

In recent years much progress has been achieved toward the development of a rigorous and meaningful quantum field theory in curved background (semiclassical gravity). In particular, in the framework of the ''algebraic approach'' (see [1] and references there) for globally hyperbolic spacetimes a complete and self-consistent description was constructed of the real scalar field obeying the Klein-Gordon equation

$$(\Box - m^2)\phi = 0. \tag{1}$$

However, there are nonglobally hyperbolic spacetimes [e.g., the Kerr black hole or spacetimes with a conical singularity (those are universes containing a cosmic string)] quantum effects in which are of obvious interest. So it would be desirable to have a theory applicable to such spacetimes as well. Unfortunately, global hyperbolicity plays a crucial role in the above mentioned theory, which therefore cannot be straightforwardly extended to the general case. The desired generalization has not been constructed so far, but a few ''reasonable candidates for minimal necessary conditions'' [1] were considered, that is, ''statements [that] begin with the phrase 'Whatever else a quantum field theory (on a given non-globally hyperbolic spacetime) consists of, it should at least involve...' '' [1]. The best-studied candidate for a necessary condition is the ''*F*-locality condition'' proposed by Kay [2]. Its importance is in that it turns out to be quite restrictive. In particular, a theorem was recently proved by Kay, Radzikowski, and Wald, which says, roughly speaking, that the *F*-locality condition cannot hold in a spacetime containing a time machine with the compactly generated Cauchy horizon [1].

The present paper is devoted to the problem of how the *F*-locality condition can be amended. The necessity of the amendments [revealed by the Kay-Radzikowski-Wald (KRW) theorem] stems from the fact that one cannot *just forbid* time machines.

It has been about six years now that a mechanism that could ''protect causality'' [3] against time machines has been actively sought. The driving force for this search is apparently the idea that the existence of a time machine would defy the usual notion of free will. This would be the case indeed if we found a paradox (like that usually called the grandfather paradox). Suppose we found such a system and its initial (that is fixed to the past of the time machine creation) state that the equations governing its evolution have no solution due to the nontrivial causal structure of the spacetime. We know that the system being prepared in this state must evolve according just to those equations (to change them we must have confessed that we overlooked some effects, which would have implied that we simply built an improper model, but not found a paradox) and at the same time we know that they have no solution. So we have to conclude that such an initial state somehow cannot be realized, that is, ''...if there are closed timelike lines to the future of a given spacelike hypersurface, the set of possible initial data for classical matter on that hypersurface... [is] heavily constrained compared [to] the same local interactions [that] were embedded in a chronology-respecting spacetime'' [4]. The dislike for such a contradiction with ''a simple notion of free will'' [5] was so strong that Rama and Sen [6], Visser [7], and in fact Hawking and Ellis [5] proposed *to postulate* the impossibility of time machines. Also a postulate prohibiting time machines is implicitly contained (as is shown by the KRW theorem) in Kay's *F*-locality condition (from now on by a ''time machine'' we mean exclusively a time machine with the compactly generated Cauchy horizon). The irony of the situation is that while no paradoxes have been found so far [8], such postulates in the absence of a mechanism that could enforce them lead to precisely the same constraints on one's will. Indeed, we know that there are initial

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conditions on the metric and the fields such that when they are fixed at a spacelike surface[1] the Einstein equations coupled with the equations of motion for these fields lead to the formation of a time machine. So if a postulate forbids time machines we only can conclude either that (i) there are some (e.g., quantum) effects which we have overlooked and which being taken into consideration always change the equations of motion so that the time machine does not form or (ii) such initial conditions are somehow forbidden. Both possibilities were considered in the literature.

(i) A popular idea was that the vacuum polarization near a would-be Cauchy horizon (when it is compactly generated) becomes so strong that its back reaction on the metric prevents the formation of the horizon. This idea, however, has never been embodied in specific results. The vacuum polarization in spacetimes with a time machine was evaluated for a few simplest cases [3,9,10][2] and it turned out that sometimes it diverges on the Cauchy horizon and sometimes it does not (in the perfect analogy with, say, the Minkowski space). So it is unlikely that this effect could always protect causality.

(ii) It is possible that initial data leading to the formation of a time machine are forbidden not by a restriction on our will but simply by the fact that they require some unrealizable conditions. It was shown [3], for example, that to create a time machine of a noncosmological nature (that is evolving from a noncompact Cauchy surface) one has to violate the weak energy condition (WEC) and a number of restrictions was found on such violations (see, e.g., [11]). None of them, however, has been able to rule time machines out. Moreover, recently a classical scenario for WEC violations was proposed [12].

Thus causality remains unprotected and any postulate prohibiting time machines without adducing a mechanism that enforces this prohibition raises the alternative of rejecting either the postulate or the idea that whether one can perform an experiment does not depend on whether causality still holds somewhere in the future.

In the case of the *F*-locality condition the alternatives at first glance seem equally unattractive since this condition is based on the *GH*-equivalence principle (see Sec. III). However, a closer inspection shows that the *F*-locality condition contains a strong arbitrary requirement (in Sec. IV we discuss this fundamental point in great detail). So one can reconcile the *GH*-equivalence principle with quantum field theory in spacetimes with a time machine by just abandoning this requirement. In doing so one still can use the GH-equivalence principle in the theory. It is only necessary to find its another mathematical implementation. As an example we consider in Sec. V the ''*MF*-locality condition.'' An important point is that while expressing the *GH*-equivalence principle (and seemingly doing it more adequately than the *F*-locality condition), *it does not forbid time machines*. From this we conclude in particular that, contrary to what was claimed in [1] and in a number of succeeding papers, the KRW theorem does not ''provide strong evidence in support of Hawking's chronology protection conjecture.'' It rather rules out the *F*-locality condition.

[1]Actually, even on a part of the surface $t=0$ of an ''almost Minkowskian'' space (cf. [3]).

[2]There are also papers where (for nonsimply connected time machines) different results based on the ''method of images'' are obtained and discussed. This method, however, involves manipulations with incurably ill-defined entities and generally allows one to obtain almost any result one wants (see [10] for a detailed discussion).

II. GEOMETRICAL PRELIMINARIES

An important role in our discussion will be played by the notion of *global hyperbolicity*. Globally hyperbolic (*GH*) spacetimes most adequately meet the concept of a ''good'' or ''usual'' spacetime (the Minkowski spacetime, for example, is *GH*).

Definition 1. A subset N of a spacetime (M,g) is called globally hyperbolic if strong causality holds in N and for any points $p,q \in N$ the set $J^+(p)\cap J^-(q)$ is compact and lies in N.

Whether or not a neighborhood $N\subset M$ is *GH* is not determined exclusively by its geometry. Due to the requirement that $[J^+(p)\cap J^-(q)]\subset N$ it may happen that N is not *GH* even though $(N,g|_N)$ is *GH* when it is regarded as a spacetime in its own right. So to describe the geometrical properties of a neighborhood proper we introduce the following.[3]

Definition 2. We call a subset N of a spacetime (M,g) intrinsically globally hyperbolic if $(N,g|_N)$ is a *GH* spacetime.

Clearly, whether a neighborhood N is an intrinsically *GH* neighborhood (*IGHN*) does not depend on the geometry of $M-N$ (in contrast to whether it is a *GHN*). To avoid confusion, note that our notion of ''global hyperbolicity'' is that of [5] and *differs* from that in [1,2]. The latter corresponds to our ''intrinsic global hyperbolicity.'' For later use let us list a few obvious properties of (intrinsically) globally hyperbolic neighborhoods [(I)*GHNs*]: (GH_1). An intersection of two (I)*GHNs* is an (I)*GHN*; GH_2, any *GHN* is an (I)*GHN* and an (I)*GHN* is a *GHN* if and only if it is causally convex (that is if and only if no causal curve leaving the (I)*GHN* returns in it). Thus intrinsic global hyperbolicity is a weaker condition than global hyperbolicity. In particular, we have the following. GH_3, for any point $P\in M$ and any its neighborhood V there exists an (I)*GHN* N such that $P\in N\subset V$, while such a *GHN* exists if and only if strong causality holds in P. Property GH_2 enables us to construct a simple and useful example of a connected (I)*GH* but not a *GH* subset of the (three-dimensional) Minkowski space.[4]

Example: A ''bad'' set. Let V be the cube $\{x_k \in(-4,4)\}$. Consider the strip $S\subset V$ (see Fig. 1) given by the system

$$x_0=\varphi/2, \quad \varphi\in[-\pi,\pi], \quad \rho\in[1,2], \tag{2}$$

[3]Connected IGH neighborhoods were called *locally causal* in [13].

[4]The existence of such a neighborhood was mentioned in [2] with reference to Penrose.

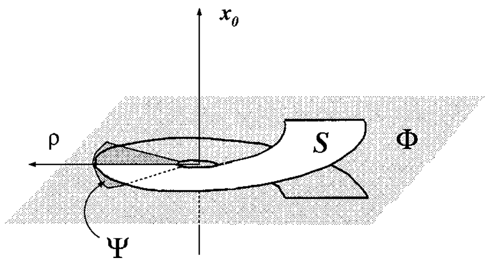

FIG. 1. Construction of a ''bad'' set.

where ρ,φ are the polar coordinates on the plane (x_1,x_2). There are causally connected points on S and, in particular, there are points connected by null geodesics lying in V (or *null related in V*, in terms of [1,2]). A simple calculation based on the fact that

$$\mathbf{A} \text{ is spacelike whenever } |A_0/A_i|<1 \tag{3}$$

shows, however, that

$$v_1,v_2\in S, \quad v_1\neq v_2, \quad v_1\preccurlyeq v_2 \Rightarrow \varphi(v_2)-\varphi(v_1)>\varphi_0>\pi.$$

So a causal curve can connect two points in S only if one of them lies above the plane $\Phi\equiv\{v|x_0(v)=0\}$ and the other below Φ. Hence (a) all causal curves connecting points of S intersect the plane Φ. Similarly, by simple though tiresome considerations one can show that (b) there is a closed set $\Theta\subset\Phi$ such that $S\cap\Theta=\varnothing$ and none of the causal curves from S to S intersects $\Psi\equiv\Phi-\Theta$. [For example, we can choose $\Psi\equiv\{v\in\Phi|\rho(v)\in(0.08,2.2),\ |\varphi(v)|<0.1\}$.] Consider now S as a subset of the spacetime $M'\equiv M-\Theta$. Properties (a) and (b) ensure that S is a (closed) achronal set and hence by Proposition 6.6.3 of [5] the interior B of its Cauchy domain in M' is a *GH* subset of M'. Thus by GH_2, B is an (I)*GHN* and not a *GHN*.

Note that we have used the fact that M is the Minkowski space only in stating Eq. (3). It can be easily seen, however, that within *any* neighborhood in *any* spacetime coordinates x_i can be found such that (3) holds in the cube $\{x_k\in(-4,4)\}$. So (being generalized to the four-dimensional case) this example proves the following proposition.

Proposition 1. For any point p and any its neighborhood V such a connected (I)*GHN* $B\subset V$ of p and such a pair of null related in V points $r,q\in B$ exist that r and q are not connected by any causal curve lying in B.

III. *F* LOCALITY

The algebraic approach to quantum field theory (below we cite only some basic points that have to do with F locality; for details see [1] and references therein) is based on the notion of the ''field algebra,'' which is a $*$ algebra with identity I generated by polynomials in ''smeared fields'' $\phi(f)$, where f ranges over the space $\mathcal{D}(M)$ of smooth real-valued functions compactly supported on M. The smeared fields $\phi(f)$ are just some abstract objects [informally they can be understood as $\phi(f)=\int_M\phi(x)f(x)\sqrt{-g}d^4x$, where $\phi(x)$ is the ''field at a point'' operator of the (nonrigorous) conventional quantum field theory]. A field algebra is defined by the relations [for all $f,h\in\mathcal{D}(M)$ and for all pairs of real numbers a,b]

$$\phi(f)=\phi(f)^*, \quad \phi(af+bh)=a\phi(f)+b\phi(h),$$
$$\phi[(\Box-m^2)f]=0 \tag{4}$$

(defining a ''prefield algebra'') and a relation fixing commutators $[\phi(f),\phi(h)]$, which we discuss in the following subsections.

Given a field algebra one can proceed to build a complete quantum theory of the free scalar field by introducing the notion of *states*, postulating some properties for ''physically realistic'' states and prescriptions for evaluating physical quantities (such as the renormalized expectation value of the stress-energy tensor) for these states. We will not go into this ''second level'' [2] of the theory.

A. The globally hyperbolic case

Definition 3. Let $\mathcal{E}$ be a subset of $\mathcal{D}(M)\times\mathcal{D}(M)$ and let $\triangle$ be a functional on pairs f,h, where $f,h\in\mathcal{D}(M)$ and $(f,h)\in\mathcal{E}$. We shall call $\triangle$ *a bidistribution on* $\mathcal{E}$ if it is separately linear and continuous (with respect to topology of $\mathcal{D}(M)$] in either variable.

To fix a commutator relation for the field algebra consider the Klein-Gordon equation (1) given on an (I)*GHN* U. Let $\triangle$ be its bidistributional solution, that is, a bidistribution on $\mathcal{D}(U)\times\mathcal{D}(U)$ satisfying $\triangle[(\Box-m^2)f,h]=\triangle[f,(\Box-m^2)h]=0$ for all $f,h\in\mathcal{D}(U)$. Among all such solutions there is a preferred one.

Definition 4. Let $\triangle_U^{A(R)}$ be the fundamental solutions of the inhomogeneous Klein-Gordon equation on a neighborhood U satisfying the property

$$\triangle_U^{A(R)}(f,g)=0 \quad \text{whenever} \quad \operatorname{supp} f\cap J^{\mp}(\operatorname{supp} g,U)=\varnothing. \tag{5}$$

Then we call a bidistributional solution of the *homogeneous* Klein-Gordon equation $\triangle_U\equiv\triangle_U^A-\triangle_U^R$ the advanced minus retarded solution on U.

It turns out that for any (I)*GHN* U, $\triangle_U$ exists and is unique. So we complete the definition of a field algebra by adding to Eq. (4) the commutator relation

$$[\phi(f),\phi(h)]=i\triangle_M(f,h)I. \tag{6}$$

Which of the bidistributional solutions of Eq. (1) is the advanced minus retarded solution for a given region U is completely determined by the causal structure of U. This allows one to prove the following important facts [2].

The F-locality property (form I). Every point p in a *GH* spacetime M has an intrinsically globally hyperbolic neighborhood U_p such that for all $f,h\in\mathcal{D}(U_p)$, relation (6) holds with $\triangle_M$ replaced by $\triangle_{U_p}$.

We can also reformulate the F-locality property in a slightly different form by ''gluing'' all these $\triangle_{U_p}$ into a single bidistribution $\triangle^F$.

Let $\triangle$ be a bidistribution on $\mathcal{E} \supset \mathcal{D}(U) \times \mathcal{D}(U)$. It induces a bidistribution $\triangle|_U$ on $\mathcal{D}(U) \times \mathcal{D}(U)$ by the rule

$$\forall f,h \in \mathcal{D}(U), \quad \triangle|_U(f,h) \equiv \triangle(f,h).$$

Definition 5. We shall call U and $\triangle$ *matching* if U is a connected $(I)GHN$ and

$$\triangle|_U = \triangle_U.$$

The F-locality property (form II). There are such an open covering of a *GH* spacetime *M* by $(I)GHNs$ $\{U_\alpha\}$ and such a bidistribution $\triangle^F$ on $\mathcal{E}_U$ that for Pr_1, $\triangle^F$ matches any U_α and for Pr_2, when $(f,h) \in \mathcal{E}_U$, relation (6) holds with $\triangle_M$ replaced by $\triangle^F$.

Here and subsequently if $\{U_\alpha\}$ is a set of neighborhoods in *M* we write $\mathcal{E}_U$ for $\cup_\alpha [\mathcal{D}(U_\alpha) \times \mathcal{D}(U_\alpha)]$.

B. The nonglobally hyperbolic case

To build a field algebra in a nonglobally hyperbolic spacetime we can start with a prefield algebra (4). Then, however, we meet a problem with the commutator relation since $\triangle_M$ is (uniquely) defined only for *GH* spacetimes and there are no obviously preferred solutions of Eq. (1) any longer. So we need some new postulate and Kay proposed [2] to infer such a postulate from the equivalence principle, which as applied to our situation he formulated as follows.

The GH-equivalence principle. On an arbitrary spacetime, the laws in the small should coincide with the ''usual laws for quantum field theory on globally hyperbolic spacetimes.''

From this principle he *postulated* in a sufficiently small neighborhood of a point in an arbitrary spacetime what holds by itself in a GH spacetime. Namely, he requires the following.

The F-locality condition (form I). Every point *p* in *M* should have an intrinsically globally hyperbolic neighborhood U_p such that, for all $f,h \in \mathcal{D}(U_p)$, relation (6) holds with $\triangle_M$ replaced by $\triangle_{U_p}$.

It is meant that a spacetime for which there is no field algebra satisfying this condition (a ''non-*F*-quantum compatible'' spacetime) cannot arise as an approximate description of a state of quantum gravity and must thus be considered as unphysical.

To reveal the logical structure of the *F*-locality condition we reformulate it analogously to the *F*-locality property.

The F-locality condition (form II). There should be such an open covering of a space-time *M* by $(I)GHNs$ $\{U_\alpha\}$ and a bidistribution $\triangle^F$ on $\mathcal{E}_U$ that for CON_1, $\triangle^F$ matches any U_α and for CON_2, when $(f,h) \in \mathcal{E}_U$, relation (6) holds with $\triangle_M$ replaced by $\triangle^F$.

An important difference between these two parts of the *F*-locality condition is that CON_2 just specifies what algebra we take to be the field algebra, while CON_1 is a nontrivial requirement placed from the outset upon the spacetime. It is significant that the proof of the KRW theorem rests upon CON_1.

The *F*-locality condition clearly does not fix *all* commutators. The value of $[\phi(f),\phi(h)]$ remains undefined for *f,h*, whose supports do not belong to a common U_α. It is more important, however, to find out whether this uncertainty extends to arbitrarily small regions. Indeed, to find such local quantities as $\langle T_{\mu\nu}\rangle(p)$ or, say, $\langle\phi^2\rangle(p)$, it would be enough to know all commutators $[\phi(f),\phi(h)]$ with functions *f,h* both supported in a small neighborhood *V* of *p*. This leads us to the following question: Is it true for at least *some* open covering $\{V_\alpha\}$ that

$$\forall (f,h) \in \mathcal{E}_V, \quad \triangle'^F(f,h) = \triangle^F(f,h) \tag{7}$$

whenever both $\triangle^F$ and $\triangle'^F$ satisfy CON_1 (with possibly different $\{U_\alpha\}$)? It turns out that the answer is negative even in the simplest case. Indeed, if *M* is the Minkowski space and $\triangle^F$ is a solution of Eq. (1) satisfying CON_1, then so is $\triangle'^F$:

$$\triangle'^F(f,g) \equiv \triangle^F(f',g) \quad \text{where} \quad f'(x^\mu) \equiv f(x^\mu) + f(x^\mu + a^\mu)$$

and by a^μ we denote an arbitrary constant spacelike vector field. Clearly, for any $\{V_\alpha\}$ we can find an a^μ such that Eq. (7) breaks down. So the *F*-locality condition was proposed only as a *necessary* condition that is to be supplemented with conditions of the second level to obtain a complete theory.

IV. THE PARADOX AND ITS RESOLUTION

The *F*-locality condition (or CON_1 to be more specific) includes actually a postulate forbidding time machines. This follows from the Kay-Radzikowski-Wald theorem.

The KRW theorem. If a spacetime has a time machine with the compactly generated Cauchy horizon, then there is no extension to *M* of the usual field algebra on the initial globally hyperbolic region *D* which satisfies the *F*-locality condition.

Here by ''the usual field algebra'' we mean an algebra where for $f,h \in \mathcal{D}(D)$ relation (6) holds with $\triangle_M$ replaced by $\triangle_D$ (for the proof of the theorem and the precise definition of *D* see [1]).

As is discussed in the Introduction, postulating causality without adducing a ''protecting'' mechanism, one comes up against a contradiction with the usual notion of free will, which can be regarded as a paradox. Such a situation (when a paradox arises from postulating in the general case a condition harmless in the *GH* case) is in no way strange or new.

Example: Classical pointlike particles. Consider a system of elastic classical balls. As long as one studies only *GH* spacetimes one sees that the following property holds.[5]

The property of balls conservation. Any Cauchy surface intersects the same number of the world lines of the balls.

Going to arbitrary spacetimes, one finds that the evolution of a system of balls is no longer uniquely fixed by what fixes it

[5]A model describing such a system can be found in [8]. A specific mathematical meaning is assigned there to the words ''a world line of a ball,'' etc. The property then can be *proved* within this model.

in the *GH* case. To overcome this problem (in the perfect analogy with the *F*-locality condition) one could adopt the following postulate[6] [note that in the general case it is just a postulate that is an *extraneous (global) condition* and not a consequence of any other local principles accepted in the model].

The condition of ball number conservation. Any partial Cauchy surface should intersect the same number of the world lines of the balls.

Then one would find [6,8] that there are ''nonclassical compatible'' spacetimes (e.g., the Deutsch-Politzer space) that are spacetimes in which initial data (i.e., data at some partial Cauchy surface) exist incompatible with the postulate of ball number conservation. This fact constitutes an (*apparent*, see [8]) paradox and so one could claim that the existence of such paradoxes suggests that time machines are prohibited [6]. On the other hand, as we discussed above, it seems more reasonable to look for contradictions which we ourselves could introduce in the model in the process of constructing. In doing so we would interpret the nonclassical compatibility of the Deutsch-Politzer spacetime as evidence not against the realizability of this spacetime, but rather against the postulate. Indeed, abandoning this postulate we resolve the paradox (and thus permit time machines) while causing no harm to any known physics [8].

The above example suggests that to avoid the difficulties connected with forbidding the time machine, which we discussed in the Introduction, it would be natural just to abandon the *F*-locality condition. The problem, however, is that while we can easily abandon the postulate of ball number conservation, the *F*-locality condition seems to be based on the philosophical bias resembling the equivalence principle, which is something one would not like to reject. So, in the remainder of the section we show that the *F*-locality condition contains actually an *arbitrary* (i.e., not implied by the *GH*-equivalence principle or any other respectable physical principle) *global requirement* and therefore can be rejected or modified without regret.

Proposition 2. For any $\triangle$ and any neighborhood V there exists a connected $(I)GHN$ $B \subset V$ that does not match $\triangle$.

Proof. Without loss of generality (see GH_3) V may be thought of as being an $(I)GHN$. So either V itself is the desired neighborhood or $\triangle|_V$ is the advanced minus retarded solution $\triangle_V$ on V. In the latter case we can simply adapt the proof of the KRW theorem [1] for our needs. Namely, let B be the set from Proposition 1 and r,q the points appearing there. To obtain a contradiction suppose that B matches $\triangle$ and hence matches $\triangle|_V = \triangle_V$ also. This would mean, by definition, that

$$(\triangle_V)|_B = \triangle_B, \tag{8}$$

but $\triangle_B(r,q) = 0$ since r and q are not causally connected in B, while $(\triangle_V)|_B$ is singular at the pair r,q (see [1] for the proof) since both of these points belong to V and are null connected in it. This is a contradiction.

Thus we see that even if a spacetime is globally hyperbolic there are two families of $(I)GHNs$ for any its point: causally convex (and thus *GH*) sets $\{G_\alpha\}$ (let us call them *good*) and those containing null related points that are intrinsically noncausally connected (we shall call them *bad* and denote by $\{B_\beta\}$). Both families include ''arbitrarily small'' sets (i.e., for any neighborhood V one can find both a ''good'' (G_{α_0}) and a ''bad'' (B_{β_0}) subsets of V). Irrespective of what meaning one assigns to the term ''the laws in the small,'' it seems reasonable to assume that they are the same for B_{β_0} and G_{α_0}. The more it is so as an observer cannot determine (by geometrical means) whether a neighborhood is good or bad without leaving it. We have seen that the good sets match the commutator function, while the bad ones do not. So it follows that the identity of physics in two sets does not imply that they both match the same bidistribution. Correspondingly, the fact that the laws in a small region coincide with any other laws does not imply that it (or any its subset) matches the commutator function on a bigger region. So *the requirement* CON_1 *that a point should have a neighborhood matching a global commutator function is not an expression of the GH-equivalence principle*, but is rather an extraneous condition. It is also an essentially *global* condition. Indeed, for any point one *always* can find a bidistribution matching *some* $(I)GHN$ of the point and so the main idea of CON_1 is that such a bidistribution should exist globally. We see thus that indeed the *F*-locality condition needs amendments since while leading to possible paradoxes it contains a strong nonjustified requirement.

V. MODIFIED *F* LOCALITY

In this section we formulate and discuss a candidate necessary condition alternative to the *F*-locality condition. Being an implementation of the *GH*-equivalence principle (coupled with the locality principle; see below), it nevertheless does not forbid any causal structure whatsoever. Thus a theory based on this condition is free from the paradoxes discussed above, which provides further evidence in favor of the idea that the existence of time machines is inconsistent not with the equivalence principle, but only with its inadequate implementation.

Consider a commutator $[\phi(x),\phi(y)]$. Physically this commutator describes the process in which a particle created from vacuum in x annihilates in y. So when we require [as we did in Eq. (6)] that the commutator function should vanish for noncausally connected x and y we just implement the (most fundamental) idea that an event can affect only those events that are connected with it by causal curves or, in other words, that particles (or information in any other form) cannot propagate faster than light. The very same idea (called locality, causality, or local causality depending on the formulation and application) suggests that if the conditions are fixed in $J^+(x) \cap J^-(y)$ (that is, in all points where a nontachionic particle propagating from x to y can find itself), then

[6] Such an approach was really developed in a number of works (e.g., see [4,6,14]).

$[\phi(x),\phi(y)]$ is thereby also fixed. Thus, from locality it seems natural to require as a necessary condition that the field algebra in a globally hyperbolic neighborhood G does not ''feel'' whether or not there is something outside G [recall that for any $x,y\in G$ any point $z\in M-G$ lies off $J^+(x)\cap J^-(y)$]. We can then construct a field algebra (at least on a part of M; see below) by adopting the following modification of the F-locality condition.

The MF-locality condition. If $\{G_\alpha\}$ is the collection of all globally hyperbolic subsets of a spacetime M, then for all $(f,h)\in\mathcal{E}_G$ relation (6) should hold with $\triangle_M$ replaced by $\triangle^{MF}$ defined to be a bidistribution on $\mathcal{E}_G$ matching each G_α.

(In other words, we require that $\phi(f)$ and $\phi(h)$ with f and h supported on a common *GHN* G_α should commute as if there were no ambient space $M-G_\alpha$ at all.) This condition obviously holds in a *GH* spacetime, where[7] $\triangle^{MF}=\triangle_M$.

Remark. In discussing the field algebra we operate with such ''nonlocal'' (by their very nature) entities as commutators $[\phi(x),\phi(y)]$, where x and y can be wide apart. No wonder that relevant statements are also formulated in nonlocal terms. In particular, both the *MF* locality and the F-locality conditions distinguish some classes of (I)*GHNs* of a point from the others. In the former case those are the causally convex neighborhoods and in the latter case the distinguished class is not specified, but its existence is postulated. However, to learn whether or not a given (I)*GHN* belongs to the distinguished class we have to consider how it is embedded in the ambient space and to take into account properties of this space [e.g., to check whether or not a set V is causally convex one must consider the whole $J^+(V)$]. In this connection we emphasize that the *MF*-locality condition is not a nonlocal postulate (much less a postulate contradicting locality). That is, it does not require that a spacetime, or a field algebra, possess any nonlocal properties. On the contrary, we found out what locality requires in a specific situation (it is the description of this situation that necessitates nonlocal terms as we argued above) and chose these requirements as a postulate of the theory.[8]

The *MF*-locality condition differs from the F-locality condition in that (a) *some IGHNs* are replaced by *each GHN* and (b) a condition is imposed only on the field algebra, but not on the geometry of the background spacetime.

Correspondingly, two important consequences take place.

(a) As we discussed in Sec. III, the F-locality condition does not uniquely fix the commutator function. Neither does the *MF*-locality condition. The situation has improved, however, in that now we can fix at least the ultraviolet behavior of the commutator function in the region $G\equiv\cup_\alpha G_\alpha$ where strong causality holds.

Proposition 3. For any spacetime M with nonempty G, $\triangle^{MF}$ exists and is unique.

Proof. Consider two *GHNs* G_i and G_k. By GH_1 their intersection is also a *GHN* (we can thus denote it G_j) and points in G_j are causally related if and only if they are causally related in G_i (and thus also in G_k). So (see [2])

$$(\triangle_{G_i})|_{G_j}=\triangle_{G_j}=(\triangle_{G_k})|_{G_j},$$

which means that we can define $\triangle^{MF}$ by the equation

$$\triangle^{MF}|_{G_\alpha}\equiv\triangle_{G_\alpha}. \tag{9}$$

This guarantees that $\triangle^{MF}$ is a desired bidistribution matching all G_α. At the same time, *any* functional matching them must satisfy Eq. (9), which proves the uniqueness. ■

(b) As we discussed above, the F-locality condition is global in nature. Either $\triangle^F$ does not exist on M and we must exclude the whole M from consideration or $\triangle^F$ exists and then no region is distinguished in this sense. The situation differs greatly if we postulate the *MF*-locality condition instead. On the one hand, *any* spacetime is allowed now [since $\triangle^{MF}$ always exists (see Proposition 3); there are no ''non-*MF*-compatible spaces''] and, on the other hand, different parts of a spacetime now have different status. Namely, each point $p\in G$ has a neighborhood U_p such that $\triangle^{MF}(f,h)$ is determined by the *MF*-locality condition at least for $f,h\in\mathcal{D}_{U_p}$. So one can develop the theory as we mentioned in the beginning of Sec. III and eventually find $\langle T_{\mu\nu}\rangle(p)$. However, this cannot be done at this stage for a point in $M-G$, where no field algebra is fixed. Thus the surface ∂G separates the area of the present version of semiclassical gravity from *terra incognita*.[9]

The role of ∂G is especially important in the time machine theory since it is ∂G where the divergence of the stress energy is expected by many authors. So it should be stressed that *physically* there is nothing particular in points of ∂G (including the ''base points''; see [1]). In perfect analogy with coordinate singularities in general relativity, ∂G does not correspond to any physical entity and the fact that we cannot find the energy density in a point of ∂G means not that it is singular or ill-defined here but simply that we do not know how to do this.

Remark. The *MF*-locality condition was proposed in this paper primarily to clarify the relation between causality violations and the *GH*-equivalence principle. However, the

[7]Generally, $\triangle^{MF}$ is not the same as $\triangle^F$. This follows directly from the nonuniqueness of $\triangle^F$ shown at the end of Sec. III.

[8]Note that the same situation takes place in the globally hyperbolic case. The postulate (6) also may seem nonlocal since the condition defining $\triangle_M$ contains [see Eq. (5)] a ''nonlocal'' part supp $f\cap J^{\mp}(\text{supp } g,U)=\varnothing$.

[9]In this respect ∂G is similar to Visser's ''reliability boundary'' [7]. The main difference is that the latter conceptually bounds the region where semiclassical gravity breaks down because of quantum gravity corrections.

uniqueness proved above and the simplicity of the underlying physical assumption suggest that perhaps it deserves a more serious consideration as a possible basis for constructing semiclassical gravity in nonglobally hyperbolic spacetimes. Then it would be interesting to find out whether the theory proposed by Yurtsever [13] (which does not, at least explicitly, appeal to any locality principle) is consistent with it.

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